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ABSTRACT

How much do contracting frictions between global firms and their suppliers impact country welfare and trade? We answer this by developing a model of global sourcing under partial contractibility, where firm-supplier relationships are exposed to a bilateral holdup problem. Sourcing decisions aggregate into a gravity equation for trade flows by organizational mode (intrafirm vs. arm's length), which we take to structural estimation. We can then evaluate welfare changes with an extended Arkolakis, Costinot and Rodriguez-Clare (2012) formula that incorporates these contracting frictions. Our counterfactual analysis reveals a sizeable average country welfare gain of 9.2% from eliminating contracting frictions in global sourcing. We further show how accounting for these frictions significantly reshapes quantitative assessments of the welfare gains from trade, including the stakes in a US-China decoupling scenario.

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1 Introduction

Global sourcing – the procurement of inputs from abroad – is a defining feature of modern production. Manufacturing firms today rely extensively on suppliers located around the world for dedicated and specialized inputs, from automotive parts, to pharmaceutical ingredients, to semiconductor chips. In fact, around three-fifths of world trade is in intermediates rather than finished goods, a share that has been relatively stable despite recent government efforts (notably in the US) to promote reshoring in the name of supply chain resilience.[1]

This rise in global sourcing has been driven by several well-studied forces: lower wage costs abroad, improvements in shipping and communications, and advances in foreign expertise in key technology segments. Ultimately too, the viability of these sourcing arrangements hinges, especially for customized inputs, on the reliability of the contracting environment in which firms and their suppliers operate. In practice, suppliers can fail to exert the effort needed to fulfill product specifications or standards that were previously agreed upon. Likewise, firms can withhold some proprietary knowhow or equipment that were promised to suppliers. Such contracting frictions afflict supply chains that are domestic in scope. Needless to say, these challenges are magnified when input sourcing cris-crosses multiple country jurisdictions and legal systems. Recognizing this, the 2020 World Development Report highlighted how weak contract enforcement remains a major impediment to countries' engagement in global value chains, affirming that "institutional quality matters" (World Bank, 2020, pp.36-37). Indeed, the strength of country rule of law is associated with better export performance (Anderson and Marcouiller, 2002); it especially facilitates specialization and exporting in contract-intensive sectors, in which firms are more exposed to holdup problems vis-à-vis their input suppliers (Levchenko, 2007; Nunn, 2007; Ranjan and Lee, 2007; Chor, 2010; Cui et al., 2021; Martin et al., 2023).[2]

This paper asks: How much do contracting frictions in global sourcing matter for country outcomes such as welfare and trade? How much can countries gain from improving the contracting environment faced by supply chain actors? How does accounting for contracting frictions alter our assessments of the welfare gains from trade, or conversely, the losses from trade decoupling? As we will see, the answers we obtain to these questions are first-order in magnitude. Yet despite their relevance to both research and policy, these aggregate implications of contracting frictions have been under-explored to date. Much existing work on global sourcing has instead pursued a firm-level perspective: For example, the canonical model of Antràs and Helpman (2004) considers firms in a North-South world with incomplete contracts, in which the choice over organizational mode (the make-or-buy decision) partially mitigates the holdup problems in firm-supplier relationships (c.f., Grossman and Hart, 1986; Hart and Moore, 1990).[3] This yields valuable insights on the foreign sourcing decisions that firms make in the shadow of contracting frictions, but leaves a gap in our understanding of their aggregate impact.

We seek in this paper to provide such an assessment. Toward this goal, we develop a model of input sourcing under partial contractibility, that we embed in a general equilibrium setting amenable to

[1] Authors' own calculations, based on the UN Broad Economic Categories (Revision 4) classification. Moreover, the input content of goods that ultimately derives from foreign sources, as computed by value added accounting methods, has risen steadily since the 1970s (Johnson and Noguera, 2012; Koopman et al., 2014; Johnson and Noguera, 2017).

[2] See Nunn and Trefler (2014) for a comprehensive survey of the literature on the role of contracting and other institutional forces in shaping country comparative advantage.

[3] See Antràs (2015) for a comprehensive synthesis of this line of work on firm-level sourcing decisions in the presence of contracting frictions, that covers both modeling developments and empirical evidence.

aggregation, estimation, and quantification. At the level of the firm, production of final goods requires inputs from a discrete number of industries; the composite input from each industry consists in turn of a large set of input varieties, modelled as a continuum following Tintelnot (2017) and Antràs et al. (2017). The firm decides the *sourcing mode* for each input variety, this being a joint decision over both the supplier's location and the organizational mode – integration or outsourcing – to adopt; in a world with J countries, there are thus $2J$ possible sourcing modes.

This sourcing activity takes place in a setting with incomplete contracts, modeled on the Grossman-Hart-Moore framework: For each input variety, both the firm and supplier perform relationship-specific input tasks. However, both parties are prone to under-invest in their tasks relative to the efficient benchmark, since not all terms of an ex-ante contract can be adjudicated upon and reliably enforced by a third-party court. The model features parameters that encode the inherent degree of contractibility of firm headquarter (respectively, supplier) tasks, as in the partial contractibility formulation of Acemoglu et al. (2007) and Antràs and Helpman (2008). In turn, the ex-post renegotiation and resulting division of surplus between the firm and each supplier are described by a separate set of bargaining share parameters. In line with the property-rights view of firm boundaries, we assume that integration confers the firm a higher bargaining share – and hence stronger incentives to invest in its input tasks – relative to an outsourcing arrangement, all else equal. Firms thus choose outsourcing when it is more important to incentivize supplier effort; on the other hand, firms integrate suppliers within ownership boundaries if retaining headquarter incentives is a greater priority.

To move from micro to macro, we build a quantitative trade model (c.f., Eaton and Kortum, 2002; Costinot et al., 2012) around this firm-level setup. Countries differ in their supplier technologies, factor prices, and iceberg trade costs. Industries differ in their factor intensities, specifically in their underlying use of headquarter versus supplier tasks. Further to these features, we allow the contractibility and bargaining parameters to vary by industry (reflecting intrinsic differences in input attributes), as well as by source and destination country (reflecting variation in institutional quality). Each firm obtains a set of $2J$ supplier draws for each input variety, one for each possible sourcing mode, from a joint technology distribution. The sourcing mode selected for an input variety is that which adds the most to the firm's overall payoff, after taking into account the supplier draws, local factor prices, trade costs, and the impact of contracting frictions on both parties' investments in input tasks. We specify the supplier draws to be from an extreme value distribution, though one that extends the standard Fréchet distribution to allow each pair of draws from the same source country to be correlated; this captures, in particular, country technological forces that are common across organizational modes. The decisions over input varieties then aggregate neatly, with the share of inputs sourced within firm boundaries from a given origin country taking on a nested logit form. As we will see, contracting frictions enter in these sourcing share expressions as iceberg-like wedges that de facto retard the supplier technologies that firms have access to.

Central to our research goal, the model delivers an expression for welfare, and consequently too for welfare changes in response to shifts in trade costs and/or contracting frictions; of note, in the limit case where all inputs are fully contractible, this nests the well-known formulae in Arkolakis et al. (2012) and Costinot and Rodríguez-Clare (2014). We can therefore evaluate counterfactuals in which trade costs are reduced, or where contracting frictions are removed, using the "hat" algebra approach from Dekle et al. (2008) and Caliendo and Parro (2015).

Our model enables a tight transition from theory to estimation. We derive a gravity equation for the value of bilateral industry trade flows by organizational mode, and hence an expression for the share of this trade that occurs within firm boundaries. This variable – the intrafirm trade share – is of special interest, as it has been used regularly since Antràs (2003) to capture the propensity towards integration in studies of firm boundary decisions.[4] While this prior empirical work has largely been reduced-form in nature, we now have a novel model-implied expression for the intrafirm trade share that we can take directly to structural estimation. The requirements for quantifying the model are fairly low: What we need is intrafirm trade share data aggregated at the country-pair-by-industry level, and functional form specifications for how key model parameters (such as the contractibility and bargaining parameters) map to relevant observables (such as country rule of law and intrinsic industry characteristics).

We implement this on US intrafirm trade shares from 2001-2005, a period of rising participation by many countries (notably, China) in global value chains. For estimation, we run a non-linear least squares procedure that minimizes the distance between model-predicted and observed intrafirm trade shares, for countries and NAICS 3-digit industries in the US Census Bureau Related-Party Trade Database; we adopt flexible functional forms for the key parameters, projecting them on second-order polynomials of the underlying observables. This yields a good fit, while validating well-known patterns, for example, that the intensity in an industry's use of headquarter services is proxied well by its capital-labor ratio (Antràs, 2003). At the same time, we find that the contractibility and bargaining parameters vary with country rule of law, and with industry measures of input specificity (Rauch, 1999) and contractibility (Nunn, 2007; Nunn and Trefler, 2008), in rich yet intuitive ways. Our estimation also confirms the quantitative relevance of the nested Fréchet correlation structure, as we would otherwise over-state the dispersion in supplier draws emanating from each country. We provide a discussion of the variation in the data that identifies our estimates. We also present sensitivity diagnostics based on Andrews et al. (2017), which provide reassurance of limited scope for bias from omitted variables.

The approach to quantification we develop leans on a fair amount of modeling structure, but the payoff from this effort is that we can now address our motivating research questions. To do so, we bring in information on initial production and trade patterns from the OECD Inter-Country Input-Output Tables, and perform a series of counterfactuals. First and foremost, we find sizeable gains from a hypothetical removal of all contracting wedges in global sourcing. On average, country welfare would be 9.2% higher in the limit case of the full contractibility world; this is substantial, being equivalent to the welfare gain from a uniform 19.3% improvement in supplier technology levels across all countries and industries. A decomposition further shows that these gains are driven by firms' improved access to foreign supplier capabilities, as well as in both firms' and suppliers' incentives to make relationship-specific investments, in the world with complete contracting.

Interestingly though, these welfare gains do not accrue monotonically when contracting frictions are gradually removed worldwide. We show that countries with weak institutional rule of law can be worse off in welfare terms with modest improvements in contracting, as high rule of law countries initially capture a larger share of global sourcing activity; this is notwithstanding the fact that all countries enjoy higher welfare when the world gets sufficiently close to full contractibility. There is thus a potential "weak institutions trap" that can prevent low rule of law countries from taking steps to coordinate

[4] See Nunn and Trefler (2008, 2013), Costinot et al. (2011), Corcos et al. (2013), Bernard et al. (2010), Defever and Toubal (2013), Antràs and Chor (2013), Díez (2014), Kohler and Smolka (2014, 2021), Kukharskyy (2016), Gorodnichenko et al. (2024), among others.

partial improvements in contracting conditions with the rest of the world.

Our quantitative model further allows us to examine how contracting conditions can interact with the classical gains from trade from lowering trade costs. Crucially, we find that assessments of the gains from trade are biased if one were to neglect contracting frictions in global sourcing; the direction of this bias is moreover not uniform across countries. For developed countries with strong rule of law (e.g., Japan, Norway), or developing countries that are already well-engaged in the global economy (e.g., China, Malaysia, Vietnam), the gains from trade are over-stated by an average of 31% when the reality of incomplete contracting is not accounted for. Put differently, the gains from trade are amplified in the full contractibility world relative to the factual world with contracting frictions, but primarily for these countries that become even more embedded in global sourcing with improvements in contracting. On the other hand, the gains from trade are under-stated for countries where the institutional rule of law is initially weak (e.g., Indonesia, Russia), by an average of 20%.

As a final counterfactual, we consider not a reduction in trade barriers, but rather a US-China decoupling scenario that speaks to recent trade tensions. Our simulations show that the welfare losses from severing bilateral trade are larger for both the US and China in a world with frictionless contracting; this is in line with the amplification of the gains from trade under full contractibility seen in the previous exercise among countries that are already well-engaged in cross-border sourcing. A stronger contracting environment would thus raise the welfare stakes of a potential US-China trade decoupling.

We offer two brief caveats on our quantitative results. First, we recover model parameters using just US data, so our counterfactuals implicitly assume that these estimated parameters provide a reasonable description too of conditions in the rest of the world. Note though that this is a data-driven constraint and not a methodological limitation, since intrafirm trade shares from other countries can be readily targeted in the estimation should comparable data become available. Second, our results should be read strictly as an evaluation of the aggregate implications of contracting frictions insofar as these operate through global sourcing, bearing in mind that there could be other channels through which contracting institutions can impact economic outcomes beyond the scope of our model.

This paper builds upon a wider literature on the global fragmentation of production and supply chains. In explaining patterns of offshoring and foreign sourcing, researchers have considered neoclassical forces related to technology and factor endowments (Feenstra and Hanson, 1996; Grossman and Rossi-Hansberg, 2008), and explored too the role of search frictions when firms need to seek out suppliers (e.g., Grossman and Helpman, 2005; Startz, 2018; Eaton et al., 2022; Grossman et al., 2024). We fit within the strand of work here that draws specifically on insights from organizational economics to analyze how contracting frictions influence these sourcing decisions (Antràs and Helpman, 2004, 2008).[5]

We contribute most directly to a growing set of studies on the aggregate implications of global sourcing and global value chains (see, for example, Yi, 2010; Antràs et al., 2017; Antràs and de Gortari, 2020; Zhou, 2020; Sposi et al., 2021; Johnson and Moxnes, 2023). This includes a small number of existing papers that size up the role of contracting frictions (Fally and Hillberry, 2018; Boehm and Oberfield, 2020; Boehm, 2022), although these have been modeled more in the tradition of the Coase-Williamson (or transactions-cost based) approach, in which the firm-supplier relationship is exposed to one-sided holdup by the input supplier. Our framework builds instead a two-sided holdup problem,

[5] In the model we develop in this paper, firms interact with a large number of suppliers, and sourcing decisions are made simultaneously. See Antràs and Chor (2013) and Alfaro et al. (2019) for property-rights models of firm boundaries where the sequentiality of the sourcing decisions plays a key role in determining organizational mode in a long supply chain.

and so will allow us to compare the importance of partial contractibility in firm headquarter versus supplier tasks. We differ too in how we take our model to data, in our use of the intrafirm trade share as the key data moment to uncover deep model parameters. Separately, there are frameworks that are more agnostic about the nature of the wedges that generate distortions from first-best actions (c.f., Hsieh and Klenow, 2009; Baqaee and Farhi, 2020); in relation to these, the goal of our paper should be seen as quantifying the impact of the particular wedges that arise from partial contractibility and bilateral holdup problems. Last but not least, we will be able with our framework to assess the role of firm boundaries, by simulating the removal of integration as a feasible organizational mode. In this regard, we connect with other work in quantitative trade that has evaluated the gains from vertical integration or from multinational production (Garetto, 2013; Ramondo and Rodríguez-Clare, 2013; Ramondo, 2014; Arkolakis et al., 2018; Alviarez, 2019), albeit with models where contracting frictions do not play a central role.

The paper proceeds as follows. Section 2 sets up the firm-level sourcing problem, which Section 3 then embeds in general equilibrium. Section 4 estimates the model's parameters, and Section 5 reports on the counterfactual exercises. We provide derivations and additional details in the online appendix.

2 Model: The Global Sourcing Decision at the Firm Level

We develop a model of global sourcing, in which each final-good producer ("firm") sources a large set of inputs (from "suppliers") in an incomplete contracting environment. We set up and analyze the firm decision problem in this section; we will then aggregate across firms in Section 3, to obtain predictions on country welfare and trade flows.

2.1 Setup

Demand: We consider a world with $J > 1$ countries. The utility of the representative consumer in country j is given by:

$$U_j = \left(\int_\omega q_j(\omega)^\rho d\omega \right)^{\frac{1}{\rho}}, \ \rho \in (0,1), \tag{1}$$

where $q_j(\omega)$ is the quantity consumed of the final-good variety ω, and the elasticity of substitution across each pair of these varieties is constant and equal to $1/(1-\rho) > 1$. Each ω is produced by a unique firm. For simplicity, we also assume that each firm produces only one final-good variety using a technology with core productivity level ϕ, where ϕ is an independent draw from a distribution described by the cdf $G_j(\phi)$, as in Melitz (2003). As all key firm-level outcomes are functions of core productivity, we will use ϕ as an alternate index for firms. Final goods are consumed in the country where they are assembled, but the inputs used are sourced globally.

The market for final goods features monopolistic competition. The utility in (1) implies that the quantity consumed is an isoelastic function of the good's price: $q_j(\phi) = A_j p_j(\phi)^{-\frac{1}{1-\rho}}$, with: $A_j = E_j P_j^{\frac{\rho}{1-\rho}}$. Here, E_j is total country expenditure on final goods, and $P_j = \left(N_j \int_\phi p_j(\phi)^{-\frac{\rho}{1-\rho}} dG_j(\phi)\right)^{-\frac{1-\rho}{\rho}}$ is the ideal price index. N_j is the mass of final-good firms in the country, so there are $N_j dG_j(\phi)$ firms with core productivity ϕ; we take N_j as exogenous and fixed in our baseline analysis, though we will later explore the case where this is pinned down via a free entry condition. With this demand structure, the firm's revenue, $R_j(\phi)$, is a concave power function of the quantity sold: $R_j(\phi) = A_j^{1-\rho} q_j(\phi)^\rho$.

Production: The structure of production is more involved, and we describe it layer by layer. The production function for a firm headquartered in country j with core productivity ϕ is:

$$y_j(\phi) = \phi \left(\prod_{k=1}^{K} \left(X_j^k(\phi) \right)^{\eta^k} \right)^{1-\alpha} L_j(\phi)^{\alpha}. \tag{2}$$

This requires: (i) material inputs from $K \geq 1$ distinct industries, where $X_j^k(\phi)$ is the *composite input* from industry $k \in \{1, \ldots, K\}$ used by the firm; and (ii) assembly labor, denoted by $L_j(\phi)$. These are combined in a Cobb-Douglas fashion (c.f., Costinot et al., 2012): $\eta^k \in (0,1)$ captures the importance of industry k in all material inputs (with $\sum_{k=1}^{K} \eta^k = 1$), while $\alpha \in (0,1)$ captures the importance of assembly labor relative to material inputs.

The industry-k composite input is itself an aggregate over a continuum of industry-specific *input varieties*, denoted by $\tilde{x}_j^k(l^k;\phi)$ and indexed by l^k. This composite input is given by:

$$X_j^k(\phi) = \left(\int_{l^k=0}^{1} \tilde{x}_j^k(l^k;\phi)^{\rho^k} dl^k \right)^{\frac{1}{\rho^k}}, \ \rho^k \in (0,1), \tag{3}$$

where ρ^k governs the degree of substitutability of the input varieties from industry k, and the measure of input varieties is normalized to 1 in each industry. This formulation of a continuum of input varieties draws on Tintelnot (2017) and Antràs et al. (2017), and will facilitate the derivation of smooth expressions for the share of inputs sourced from each country under each organizational mode.

Each input variety in (3) is in turn a Cobb-Douglas combination of headquarter services, $h_j^k(l^k;\phi)$, and supplier services, $x_j^k(l^k;\phi)$, as in Antràs (2003):

$$\tilde{x}_j^k(l^k;\phi) = \left[h_j^k(l^k;\phi) \right]^{\alpha^k} \left[x_j^k(l^k;\phi) \right]^{1-\alpha^k}. \tag{4}$$

As the names suggest, the input services embodied in $h_j^k(l^k;\phi)$ are provided by the firm headquarters (e.g., design or managerial inputs), while $x_j^k(l^k;\phi)$ is obtained from the supplier whom the firm has contracted with for l^k (e.g., parts and components). We refer to $\alpha^k \in (0,1)$ as the headquarter-intensity of industry k.

In the final layer of the production function, we incorporate differences in the inherent degree of contractibility of headquarter and supplier services. Drawing on Acemoglu et al. (2007) and Antràs and Helpman (2008), $h_j^k(l^k;\phi)$ and $x_j^k(l^k;\phi)$ are each composed of a unit continuum of "tasks", indexed by ι_h and ι_x respectively, with:

$$h_j^k(l^k;\phi) = \exp\left\{ \int_{\iota_h=0}^{\mu_{hij}^k} \log h_j^k(\iota_h;\phi,l^k) d\iota_h + \int_{\iota_h=\mu_{hij}^k}^{1} \log h_j^k(\iota_h;\phi,l^k) d\iota_h \right\}, \text{ and} \tag{5}$$

$$x_j^k(l^k;\phi) = \exp\left\{ \int_{\iota_x=0}^{\mu_{xij}^k} \log x_j^k(\iota_x;\phi,l^k) d\iota_x + \int_{\iota_x=\mu_{xij}^k}^{1} \log x_j^k(\iota_x;\phi,l^k) d\iota_x \right\}. \tag{6}$$

For $x_j^k(l^k;\phi)$, all supplier tasks in the full measure $\iota_x \in [0,1]$ are performed by the unique supplier that the firm has selected for that input variety l^k. Among these, the tasks in the range $\iota_x \in [0, \mu_{xij}^k]$ are fully contractible in that the supplier effort levels $x_j^k(\iota_x;\phi,l^k)$ can be specified and enforced subsequently by an independent third-party, such as a court of law. The remaining tasks in $(\mu_{xij}^k, 1]$ are noncontractible. Likewise for $h_j^k(l^k;\phi)$, all headquarter tasks ι_h indexed on the unit interval are provided by the firm.

The headquarter investment levels $h_j^k(\iota_h; \phi, l^k)$ for tasks $\iota_h \in [0, \mu_{hij}^k]$ are fully contractible, while those in $(\mu_{hij}^k, 1]$ are noncontractible.

Both headquarter and supplier tasks are relationship-specific in that they are tailored to the requirements of this particular firm. Thus, $h_j^k(\iota_h; \phi, l^k)$ and $x_j^k(\iota_x; \phi, l^k)$ have a diminished value – which we normalize to zero – if one were to attempt to use them in final-good varieties for which they have not been customized. This exposes the production process to a bilateral holdup problem, as in Grossman and Hart (1986) and Hart and Moore (1990): Should either the headquarters or the supplier withhold delivery of their tasks, this would nullify the other party's payoff from undertaking the relationship-specific effort.[6] What distinguishes contractible from noncontractible tasks is that the investment levels for the former can be assured by spelling these out in the initial contract, whereas the firm headquarters (respectively, supplier) has full discretion over how much effort to invest in the latter.

We therefore refer to μ_{hij}^k and μ_{xij}^k as "contractibility parameters", since the degree to which headquarter and supplier services can be contracted upon is increasing in μ_{hij}^k and μ_{xij}^k respectively. As the notation indicates, we allow these parameters to vary across input industries k, since the ease with which contracts can be written and enforced could well depend on physical and technological characteristics tied to the inputs. We are also intentionally flexible in allowing μ_{hij}^k and μ_{xij}^k to vary with the source (i) and destination (j) countries, to reflect country conditions – such as legal institutions – that are relevant for the discharge of commercial contracts.[7] Note that all inputs are fully contractible when $\mu_{hij}^k = \mu_{xij}^k = 1$ for all i, j, and k; this is the special case we will simulate later, in order to gauge the aggregate effects of contracting frictions relative to this frictionless world.

This wraps up our description of the production function. To recap, the structure of production is akin to a "spider" (Baldwin and Venables, 2013): Final goods are put together from composite industry inputs (and assembly labor) that are sourced simultaneously. Each composite input is a CES aggregate of input varieties, that themselves embody headquarter and supplier services, which are each composed of contractible and noncontractible tasks. The assembled final goods are sold and consumed domestically.

2.2 Input Sourcing Environment

We next set up the firm's sourcing decision. For each l^k, this entails a decision over both location and organizational mode: The input variety can be obtained from any of the J countries in the world (including the home country). Within each country, it can be procured from a supplier that is integrated within the firm's ownership boundaries (*integration*, denoted by V) or from a supplier that the firm transacts with at arm's length (*outsourcing*, denoted by O).[8]

[6] From (5) and (6), should the supplier fail to deliver input services – so that $x_j^k(\iota_x; \phi, l^k) = 0$ – for a measurable subset of the tasks ι_x, this would result in $x_j^k(l^k; \phi)$ and hence $\tilde{x}_j^k(l^k; \phi)$ having zero value, so that the input variety l^k would make no contribution to the composite input $X_j^k(\phi)$ and to incremental revenue (as defined in equation (7)). One interpretation here is that the production process can still be completed with generic input services used *in lieu*, but this automatically results in l^k contributing zero value. An analogous statement applies should the firm withhold its input services.

[7] To be clear, the identity of the source country i from which a given input variety l^k is procured is an endogenous decision of the firm; we have suppressed this dependence of i on l^k to keep the notation neater. To be fully formal too, the subscripts of $h_j^k(l^k; \phi)$ and $x_j^k(l^k; \phi)$ should feature a further i index, which we have omitted for brevity.

[8] While there is variation in practice in the extent of the ownership stake that firms might take in their suppliers, we follow the literature in modeling this choice between integration and outsourcing as a binary decision. This is consistent with the view that crossing a critical threshold in terms of ownership share would confer on the firm substantive residual rights of control over the supplier in question. On a related note, the literature has also considered richer organizational

Contracting and Bargaining: There is a large pool of identical potential suppliers in each country. However, once the firm has picked a particular supplier for an input variety l^k, the relationship-specific nature of the tasks that both parties undertake fundamentally transforms a situation of ex-ante competition among potential suppliers into one where the chosen supplier and the firm are ex-post "locked in" (Williamson, 1985): The supplier can threaten to render the firm's investments in headquarter tasks worthless by withholding its input services, and vice versa.

The payoffs in any given firm-supplier relationship are thus determined in ex-post renegotiation. We adopt a generalized Nash bargaining protocol, in which the surplus generated by the relationship is divided between the two parties, and where the parameters that govern this split depend (among other things) on the organizational mode. Let β^k_{ijV} denote the Nash bargaining share that accrues to a country-j firm if it integrates an industry-k supplier from country i, and let β^k_{ijO} denote the share the firm obtains instead under outsourcing. (The shares received by the supplier are thus $1-\beta^k_{ijV}$ and $1-\beta^k_{ijO}$ respectively.) Following Grossman and Hart (1986), we assume that: $0 < \beta^k_{ijO} < \beta^k_{ijV} < 1$, for any given i, j, and k, so the division of surplus is tilted in the firm's favor under integration; this reflects the firm's ability to exercise residual control rights and recover more value should a breakdown in the relationship necessitate this. Integration thus confers the firm with greater incentives to invest in headquarter tasks, whereas outsourcing will better elicit the supplier's effort in its input tasks.

The bargaining shares, β^k_{ijV} and β^k_{ijO}, can further vary by industry, as well as by source and destination country. Features inherent to inputs from an industry – such as whether these are relatively homogeneous (Rauch, 1999) – would in principle affect how much pecuniary value the firm can recover from the open market should the relationship with the supplier break down (c.f., Antràs, 2015; Eppinger and Kukharskyy, 2021). Also, both source and destination country conditions can matter for bargaining outcomes; in countries with stronger rule of law, these institutional protections can help to preserve the value of inputs over which the firm has control rights, by for example restraining the supplier from taking unilateral actions to destroy the inputs amid a dispute.

Timing: Sourcing and production proceed as follows. We refer to the tuple (i, χ) as the "sourcing mode" for an input variety, if this is obtained from a supplier in country $i \in \{1, \dots, J\}$ while adopting organizational mode $\chi \in \{V, O\}$.

- Prior to any contracting or production, the firm observes the full set of draws that governs its marginal costs for sourcing. This comprises $2J$ draws for each input variety, corresponding to the $2J$ possible sourcing modes. (A separate iid set of $2J$ draws is obtained for each l^k.) We will specify the distribution that governs these draws in equation (10) below.
- After observing its draws, the firm chooses the optimal sourcing mode for each input variety. For each l^k, the firm posts a contract with: (i) an ex-ante participation fee; (ii) the chosen sourcing mode (i, χ); and (iii) the investment levels $h^k_j(\iota_h; \phi, l^k)$ and $x^k_j(\iota_x; \phi, l^k)$ for the contractible tasks $\iota_h \in [0, \mu^k_{hij}]$ and $\iota_x \in [0, \mu^k_{xij}]$ respectively. These are the only binding terms in the contract; attempts to spell out investment levels for noncontractible tasks are moot, since these are not verifiable and enforceable by a third-party. (A separate contract is posted for each l^k.)
- Suppliers apply for these posted contracts, and the firm picks one supplier for each l^k. The firm is indifferent as to the actual identity of the supplier, since all potential suppliers are ex-ante

structures, such as outsourcing with delegation of authority over the sourcing of inputs (see Karabay, 2022).

identical. Moreover, the ex-ante competition among potential suppliers means that the firm fully extracts the ex-post surplus via the participation fee.

- The firm transfers the draw for the optimal sourcing mode for l^k to the chosen supplier. The supplier for l^k discretionarily chooses how much to invest in the noncontractible supplier tasks, i.e., $x_j^k(\iota_x; \phi, l^k)$ for $\iota_x \in (\mu_{xij}^k, 1]$, while the firm chooses how much to invest in its noncontractible headquarter tasks, i.e., $h_j^k(\iota_h; \phi, l^k)$ for $\iota_h \in (\mu_{hij}^k, 1]$. The supplier and the firm also make the investments for $\iota_x \in [0, \mu_{xij}^k]$ and for $\iota_h \in [0, \mu_{hij}^k]$, as prescribed in the initial contract. All these investments in tasks across all input varieties occur at the same time.
- Upon delivery of the headquarter and supplier tasks, the firm bargains bilaterally with the supplier for l^k over the incremental revenue contributed by the input variety.
- Each input variety is put together from the task inputs following (4). Each composite input is put together from the $\tilde{x}_j^k(l^k; \phi)$'s following (3). The composite inputs $X_j^k(\phi)$ are combined with assembly labor $L_j(\phi)$ based on (2) to produce the final good.

As outlined above, the firm engages in simultaneous bilateral bargaining with each of its chosen suppliers. We adopt a "Nash-in-Nash" protocol for this process (c.f., Horn and Wolinsky, 1988): the firm's bargaining with the supplier for l^k occurs taking as given the outcomes of the firm's interactions with the suppliers for all other input varieties from all industries.[9] The firm and the supplier in question bargain over the *incremental revenue* – or more precisely, the incremental contribution to the firm's overall revenue – generated by l^k. We derive an expression for this incremental revenue with a continuum of input varieties drawing on the approach in Acemoglu et al. (2007): we first work out the contribution attributable to a single supplier when there are $\mathcal{L}$ identical suppliers each responsible for a measure $1/\mathcal{L}$ of the input varieties, and then take the limit as $\mathcal{L} \longrightarrow \infty$ (i.e., the measure of input varieties from each supplier becomes infinitesimally small). As we show in Appendix A.1, this yields the following expression for the incremental revenue associated with l^k:

$$r_j^k(l^k; \phi) = (1-\alpha)\frac{\rho\eta^k}{\rho^k} R_j(\phi) \left(\frac{\tilde{x}_j^k(l^k; \phi)}{X_j^k(\phi)} \right)^{\rho^k}. \tag{7}$$

The bilateral surplus which the firm and the supplier for l^k bargain over thus constitutes a share $(1-\alpha)\frac{\rho\eta^k}{\rho^k}(\tilde{x}_j^k(l^k; \phi)/X_j^k(\phi))^{\rho^k}$ of the revenue, $R_j(\phi)$, from the final good. It is straightforward to see that this share of total revenue: (i) increases in the importance of the industry k in production, as captured by the Cobb-Douglas exponent $(1-\alpha)\eta^k$; and: (ii) increases in the importance of l^k in the industry-k composite, as captured by $\tilde{x}_j^k(l^k; \phi)/X_j^k(\phi)$. Moreover, if $\tilde{x}_j^k(l^k; \phi)/X_j^k(\phi) < 1$ and so the input variety l^k is not too important, this share of $R_j(\phi)$ that the two parties bargain over: (iii) decreases in ρ^k, as it becomes easier to substitute for l^k with other industry-k input varieties.[10]

[9] This "Nash-in-Nash" solution concept has been adopted extensively in industrial organization to analyze bilateral oligopoly settings (i.e., where there are multiple players on both sides of the market who exert market power). Recent applications in the trade literature include Bagwell et al. (2020, 2021) who study tariff bargaining, and Grossman et al. (2024) who study sourcing decisions in the presence of search frictions. We implicitly assume that it is too costly for a sufficiently large number of suppliers who constitute a measurable subset of the unit interval to collude when bargaining with the firm. See Schwarz and Suedekum (2014) for an alternative formulation of the bargaining protocol via a Shapley value approach, that also accommodates the co-existence of both integrated and arm's-length suppliers.

[10] Note that aggregating up the incremental revenue in (7) across all input varieties from all industries k, we have:

$$\sum_{k=1}^{K}(1-\alpha)\frac{\rho\eta^k}{\rho^k} R_j(\phi) \int_{l^k=0}^{1} \left(\frac{\tilde{x}_j^k(l^k; \phi)}{X_j^k(\phi)} \right)^{\rho^k} dl^k = \left(\sum_{k=1}^{K}(1-\alpha)\frac{\rho\eta^k}{\rho^k} \right) R_j(\phi).$$

2.3 Input Task Decisions

We now solve for the optimal task investments – i.e., the $h_j^k(\iota_h;\phi,l^k)$'s and $x_j^k(\iota_x;\phi,l^k)$'s – via backward induction, based on the timing of events described above.

Noncontractible tasks: For each firm and supplier, we first solve for their respective discretionary effort investments in noncontractible tasks; each decision problem here takes as given the noncontractible investments of the other party, as well as the levels of contractible task effort and the sourcing mode that were written into the initial contract.

Specifically, the firm chooses $h_j^k(\iota_h;\phi,l^k)$ for all $\iota_h \in (\mu_{hij}^k, 1]$ in order to maximize:

$$\beta_{ij\chi}^k r_j^k(l^k;\phi) - s_j \int_{\mu_{hij}^k}^1 h_j^k(\iota_h;\phi,l^k)d\iota_h, \tag{8}$$

this being the share $\beta_{ij\chi}^k$ of the incremental revenue the firm obtains from its bargaining with the supplier for l^k, net of the cost of providing these noncontractible headquarter tasks. Here, s_j is the cost the firm bears per unit of $h_j^k(\iota_h;\phi,l^k)$; this cost is incurred in the firm's home country in units of capital, a factor of production used exclusively for headquarter tasks.

The supplier in turn receives the remaining $1-\beta_{ij\chi}^k$ share of the incremental revenue. This supplier chooses the noncontractible task levels, $x_j^k(\iota_x;\phi,l^k)$ for all $\iota_x \in (\mu_{xij}^k, 1]$, to maximize:

$$(1-\beta_{ij\chi}^k)r_j^k(l^k;\phi) - c_{ij\chi}^k(l^k;\phi)\int_{\mu_{xij}^k}^1 x_j^k(\iota_x;\phi,l^k)d\iota_x, \tag{9}$$

where $c_{ij\chi}^k(l^k;\phi)$ is the unit cost of these supplier tasks under sourcing mode (i,χ); these are performed in country i by labor, with an associated wage equal to w_i. Although individual firms and suppliers take w_i and s_j as given, these factor prices are determined endogenously in general equilibrium.

Supplier technology and costs: The supplier unit cost is given by: $c_{ij\chi}^k(l^k;\phi) = d_{ij}^k w_i / Z_{ij\chi}^k(l^k;\phi)$. Here, $Z_{ij\chi}^k(l^k;\phi)$ is the inverse unit labor requirement of supplier tasks for l^k when these are sourced from country i under organizational mode χ.[11] Iceberg trade costs $d_{ij}^k \geq 1$ are incurred to deliver the inputs from i to destination country j; we set $d_{jj}^k = 1$ for all j as a normalization.

The $Z_{ij\chi}^k(l^k;\phi)$'s themselves are obtained from an underlying technology distribution: For each l^k, the firm receives a set of draws for the $2J$ possible sourcing modes (i,χ). Each set of $2J$ draws is independently taken from a "nested Fréchet" distribution with joint cdf:

$$Pr\left(Z_{1jV}^k \leq z_{1jV}^k, Z_{1jO}^k \leq z_{1jO}^k, \ldots, Z_{JjO}^k \leq z_{JjO}^k\right) = \exp\left\{-\sum_{i=1}^J T_i^k \left((z_{ijV}^k)^{-\frac{\theta^k}{1-\lambda_i}} + (z_{ijO}^k)^{-\frac{\theta^k}{1-\lambda_i}}\right)^{1-\lambda_i}\right\}, \tag{10}$$

where $T_i^k > 0$, $\theta^k > 1$ and $0 < \lambda_i < 1$. The T_i^k's are scale parameters, with a larger T_i^k associated with a lower mean unit labor requirement for source country i in industry k. In turn, $\theta^k > 1$ is the shape parameter, which is inversely related to the dispersion of the draws.

We thus require that: $\sum_{k=1}^K (1-\alpha)\frac{\rho\eta^k}{\rho^k} < 1$, to ensure that the payments to input suppliers do not fully exhaust the revenue of the firm. This condition is readily satisfied in practice when we take our model to quantification.

[11] We index $Z_{ij\chi}^k(l^k;\phi)$ by ϕ merely to reflect that the inverse unit labor requirement is tied to the identity of the final-good producer; the nested Fréchet distribution in (10) from which $Z_{ij\chi}^k(l^k;\phi)$ is drawn does not depend on the core productivity of the firm in question.

The cdf in (10) extends the conventional Fréchet distribution from Eaton and Kortum (2002) to accommodate a correlation structure among the realized draws: There are J country "nests", and the λ_i parameter induces a correlation between the two draws – for the organizational modes $\chi = V$ and $\chi = O$ – from the country-i nest (for $i \in \{1, \ldots, J\}$).[12] The higher is λ_i, the more closely aligned are these two draws; in fact, the limit case $\lambda_i = 1$ implies identical unit costs under both integration and outsourcing. If on the other hand $\lambda_i = 0$ for all i, (10) reduces to a situation where the $2J$ draws are each from independent Fréchet distributions with cdf: $\exp\{-T_i^k (z_{ij\chi}^k)^{-\theta^k}\}$. In this latter case, the decision over sourcing mode would feature the independence of irrelevant alternatives (IIA), wherein the probability that a given sourcing mode is optimal relative to another would not depend on any third options. This property is relaxed when $\lambda_i > 0$: It allows the share of firms that outsource from say China relative to the share that outsource from Vietnam to (in principle) be affected by the share that procures the input under integration from China; this is due to the correlation in the two China draws, that reflects the common technology base (T_i^k) from which they emanate.[13] As we will see, this nested Fréchet distribution yields tractable expressions for aggregate trade flows and welfare, even while it enriches the sourcing problem at the micro level. In our quantitative work, we will in fact obtain estimates of the λ_i's that are strictly positive; failure to account for this correlation would thus bias the size of the welfare gains that we infer from removing contracting frictions.

Contractible tasks: The firm's problem in (8) and the supplier's problem in (9) yield expressions for the noncontractible investments – $h_j^k(\iota_h; \phi, l^k)$ for $\iota_h \in (\mu_{hij}^k, 1]$ and $x_j^k(\iota_x; \phi, l^k)$ for $\iota_x \in (\mu_{xij}^k, 1]$ – that are functions of the terms of the initial contract, including the pre-specified contractible task levels and the sourcing mode. We now back up to the preceding stage of the bilateral interaction, and solve for the contractible task levels – for $\iota_h \in [0, \mu_{hij}^k]$ and $\iota_x \in [0, \mu_{xij}^k]$ – that will be written into the posted contract. Bearing in mind that the firm extracts the supplier's ex-post payoff in (9) via the ex-ante participation fee, these contractible task levels are set in the initial contract by the firm to maximize the total surplus, $F_{ij\chi}^k(l^k; \phi)$, generated by this firm-supplier relationship:

$$F_{ij\chi}^k(l^k; \phi) = r_j^k(l^k; \phi) - s_j \int_0^1 h_j^k(\iota_h; \phi, l^k) d\iota_h - c_{ij\chi}^k(l^k; \phi) \int_0^1 x_j^k(\iota_x; \phi, l^k) d\iota_x. \tag{11}$$

This is done taking as given the anticipated investments in noncontractible task effort that will subsequently be made.[14]

Solution for input task levels: We focus on a symmetric equilibrium that features a uniform investment level $h_{cj}^k(l^k; \phi)$ across contractible headquarter tasks (i.e., $h_j^k(\iota_h; \phi, l^k) = h_{cj}^k(l^k; \phi)$ for all $\iota_h \in [0, \mu_{hij}^k]$), and a separate common investment level $h_{nj}^k(l^k; \phi)$ across noncontractible headquarter tasks ($\iota_h \in (\mu_{hij}^k, 1]$); analogously on the supplier side, we denote the uniform investment in contractible and noncontractible tasks respectively by $x_{cj}^k(l^k; \phi)$ for $\iota_x \in [0, \mu_{xij}^k]$ and $x_{nj}^k(l^k; \phi)$ for $\iota_x \in (\mu_{xij}^k, 1]$.[15] As we show in Appendix A.1, and similar to Antràs and Helpman (2008), we obtain:

[12] To be clear, while the $2J$ draws for a given input variety l^k exhibit a within-nest correlation, the draws across any pair of input varieties l^k and $(l^k)' \neq l^k$ are independent sets of draws from (10).

[13] Equation (10) is the Fréchet analogue of the nested logit structure often adopted to relax the IIA property inherent in multinomial logit models; the joint cdf of the log $Z_{ij\chi}^k(l^k; \phi)$'s is precisely a nested logit distribution. Other papers that work with Fréchet distributions with within-nest correlations include: Lagakos and Waugh (2013), Ramondo and Rodríguez-Clare (2013), Arkolakis et al. (2018), Brandt et al. (2021), Farrokhi and Pellegrina (2023), among others. See also Lind and Ramondo (2023) for a more general treatment of correlated technology draws in quantitative trade models.

[14] We substitute the noncontractible task investments solved for from (8) and (9) into the $F_{ij\chi}^k(l^k; \phi)$ maximand in (11), before taking the first-order conditions for the contractible task investments; see Appendix A.1 for details.

[15] In Appendix A.1, we show that it is in fact a best response for the firm (respectively, supplier) to enact a uniform

$$\left.\begin{array}{ll} h_{cj}^k(l^k;\phi) = \frac{\alpha^k}{s_j}\frac{\rho^k}{1-\rho^k}(\Xi_{ij\chi}^k)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} Z_{ij\chi}^k(l^k;\phi)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}, & h_{nj}^k(l^k;\phi) = \beta_{ij\chi}^k \frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k} h_{cj}^k(l^k;\phi) \\ x_{cj}^k(l^k;\phi) = \frac{1-\alpha^k}{d_{ij}^k w_i}\frac{\rho^k}{1-\rho^k}(\Xi_{ij\chi}^k)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} Z_{ij\chi}^k(l^k;\phi)^{\frac{1-\rho^k\alpha^k}{1-\rho^k}}, & x_{nj}^k(l^k;\phi) = (1-\beta_{ij\chi}^k)\frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k} x_{cj}^k(l^k;\phi) \end{array}\right\}, \quad (12)$$

where we have collected various terms in $\zeta_{ij\chi}^k$, ζ_{ij}^k, and $\Xi_{ij\chi}^k$, defined as follows:

$$\zeta_{ij\chi}^k = 1 - \rho^k\alpha^k(1-\mu_{hij}^k)\beta_{ij\chi}^k - \rho^k(1-\alpha^k)(1-\mu_{xij}^k)(1-\beta_{ij\chi}^k), \quad (13)$$

$$\zeta_{ij}^k = 1 - \rho^k\alpha^k(1-\mu_{hij}^k) - \rho^k(1-\alpha^k)(1-\mu_{xij}^k), \text{ and} \quad (14)$$

$$\Xi_{ij\chi}^k = \left(\frac{(1-\alpha)\rho\eta^k R_j(\phi)}{(X_j^k(\phi))^{\rho^k}}\right)^{\frac{1}{\rho^k(1-\alpha^k)}} \left(\frac{1-\rho^k}{\rho^k}\right)^{\frac{1-\rho^k}{\rho^k(1-\alpha^k)}} \left(\frac{\alpha^k}{s_j}\right)^{\frac{\alpha^k}{1-\alpha^k}} \left(\frac{1-\alpha^k}{d_{ij}^k w_i}\right) B_{ij\chi}^k, \text{ with} \quad (15)$$

$$B_{ij\chi}^k = (\zeta_{ij\chi}^k/\zeta_{ij}^k)^{\frac{\zeta_{ij}^k}{\rho^k(1-\alpha^k)}} (\beta_{ij\chi}^k)^{\frac{\alpha^k}{1-\alpha^k}(1-\mu_{hij}^k)} (1-\beta_{ij\chi}^k)^{1-\mu_{xij}^k}. \quad (16)$$

The input task levels in (12) are functions of factor prices (w_i, s_j), as well as of the parameters that govern production (ϕ, α, η^k, ρ^k, α^k), final-good demand (ρ), trade costs (d_{ij}^k), the contractibility of inputs (μ_{hij}^k, μ_{xij}^k), and bargaining ($\beta_{ij\chi}^k$).[16] These expressions bear sensible properties. We have: $h_{cj}^k(l^k;\phi)/x_{cj}^k(l^k;\phi) = (\alpha^k/(1-\alpha^k))(c_{ij\chi}^k(l^k;\phi)/s_j)$ and $h_{nj}^k(l^k;\phi)/x_{nj}^k(l^k;\phi) = (\beta_{ij\chi}^k/(1-\beta_{ij\chi}^k))(h_{cj}^k(l^k;\phi)/x_{cj}^k(l^k;\phi))$, so for both contractible and noncontractible tasks, the effort invested by the firm relative to the supplier: (i) increases in the headquarter-intensity α^k of the input; but: (ii) decreases in the relative unit cost $s_j/c_{ij\chi}^k(l^k;\phi)$. Moreover, for noncontractible tasks specifically, the investment by the firm relative to the supplier: (iii) rises in the firm's Nash bargaining share $\beta_{ij\chi}^k$ (holding all else constant).

Since $\beta_{ij\chi}^k \in (0,1)$ and $\zeta_{ij}^k/\zeta_{ij\chi}^k \leq 1$, one can see from (12) that each party's contractible investments weakly exceed that in their noncontractible tasks, i.e., $h_{cj}^k(l^k;\phi) \geq h_{nj}^k(l^k;\phi)$ and $x_{cj}^k(l^k;\phi) \geq x_{nj}^k(l^k;\phi)$. Of note, one can further show that $h_{cj}^k(l^k;\phi)$, $x_{cj}^k(l^k;\phi)$, $h_{nj}^k(l^k;\phi)$, and $x_{nj}^k(l^k;\phi)$ are all increasing functions in both the contractibility parameters, μ_{hij}^k and μ_{xij}^k.[17] In fact, when $\mu_{hij}^k = \mu_{xij}^k = 1$, the expressions for $h_{cj}^k(l^k;\phi)$ and $x_{cj}^k(l^k;\phi)$ are equal to the respective first-best effort levels that would be chosen in the absence of any contracting frictions.[18] This underscores the usefulness of this formulation of partial input contractibility: Reductions in contracting frictions are captured by increases in μ_{hij}^k and μ_{xij}^k, with the limit case where $\mu_{hij}^k = \mu_{xij}^k = 1$ for all i, j, and k being the full contractibility world.

2.4 Sourcing mode choice

We now characterize the optimal sourcing mode for the procurement of input varieties. Substituting from (12) into (11), the total surplus from the bilateral relationship with the supplier of l^k can be written

investment level across its noncontractible tasks, and that it is in turn optimal to specify a uniform investment level across all firm (respectively, supplier) contractible tasks in the initial contract.

[16] More formally, $h_{cj}^k(l^k;\phi)$ depends not only on the identity of the input variety l^k, but also on the sourcing mode (i,χ) that is chosen for l^k. To streamline the notation, we have not made this dependence on (i,χ) explicit in the arguments of $h_{cj}^k(l^k;\phi)$, nor analogously in the arguments of $h_{nj}^k(l^k;\phi)$, $x_{cj}^k(l^k;\phi)$, and $x_{cj}^k(l^k;\phi)$.

[17] This stems from the fact that $\zeta_{ij}^k/\zeta_{ij\chi}^k$ and $\Xi_{ij\chi}^k$ are increasing in both μ_{hij}^k and μ_{xij}^k. For $\zeta_{ij}^k/\zeta_{ij\chi}^k$, this follows from the definitions in (13) and (14), and an application of the quotient rule. As for $\Xi_{ij\chi}^k$, this is a consequence of $B_{ij\chi}^k$ being increasing in both μ_{hij}^k and μ_{xij}^k, which we establish in Lemma 1 in Section 2.4. (See Appendix A.1 for the detailed proofs.) Note too that $h_{nj}^k(l^k;\phi)/h_{cj}^k(l^k;\phi)$ and $x_{nj}^k(l^k;\phi)/x_{cj}^k(l^k;\phi)$ are both increasing in μ_{hij}^k and μ_{xij}^k; reductions in contracting frictions are thus associated with a greater propensity to invest in noncontractible relative to contractible tasks.

[18] In particular, this follows from the fact that $\zeta_{ij}^k/\zeta_{ij\chi}^k \leq 1$, with equality holding if $\mu_{hij}^k = \mu_{xij}^k = 1$.

compactly as: $F^k_{ij\chi}(l^k;\phi) = (\Xi^k_{ij\chi} Z^k_{ij\chi}(l^k;\phi))^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}$ when this occurs under sourcing mode (i,χ), where $\Xi^k_{ij\chi}$ was defined above in (15). (See Appendix A.1 for the derivation.) Recall that the firm receives this total surplus after the supplier's participation fee is taken into account. The organizational mode selected by the firm will thus seek to maximize $F^k_{ij\chi}(l^k;\phi)$, by balancing out as best as possible the incentives of the two parties to invest in their respective noncontractible tasks.

More formally, the optimal sourcing mode for l^k is given by:

$$\arg\max_{(i',\chi')} \; \Xi^k_{i'j\chi'} Z^k_{i'j\chi'}(l^k;\phi). \tag{17}$$

With the $Z^k_{i'j\chi'}(l^k;\phi)$'s drawn from the nested Fréchet cdf in (10), we can accordingly compute the probability, $\pi^k_{ij\chi}$, that the solution to (17) is (i,χ). By the law of large numbers, $\pi^k_{ij\chi}$ is then also the share of input varieties for which (i,χ) is the chosen sourcing mode. In Appendix A.1, we show that:

$$\pi^k_{ij\chi} = \pi^k_{ij}\pi^k_{\chi|ij}. \tag{18}$$

The sourcing probability $\pi^k_{ij\chi}$ takes a form familiar from nested logit models of discrete choice. It can be decomposed as the product of: (i) π^k_{ij}, the probability that an industry-k input variety is sourced from country i; and: (ii) $\pi^k_{\chi|ij}$, the probability that the organizational mode is χ, conditional on selecting i as the source country. These latter two probabilities are given explicitly by:

$$\pi^k_{ij} = \frac{T^k_i (d^k_{ij} w_i)^{-\theta^k} (B^k_{ij})^{\theta^k}}{\sum_{i'=1}^J T^k_{i'} (d^k_{i'j} w_{i'})^{-\theta^k} (B^k_{i'j})^{\theta^k}}, \text{ and } \; \pi^k_{\chi|ij} = \frac{(B^k_{ij\chi})^{\frac{\theta^k}{1-\lambda_i}}}{(B^k_{ijV})^{\frac{\theta^k}{1-\lambda_i}} + (B^k_{ijO})^{\frac{\theta^k}{1-\lambda_i}}}, \tag{19}$$

where $B^k_{ij\chi}$ was defined in (16), and $B^k_{ij} = ((B^k_{ijV})^{\frac{\theta^k}{1-\lambda_i}} + (B^k_{ijO})^{\frac{\theta^k}{1-\lambda_i}})^{\frac{1-\lambda_i}{\theta^k}}$ aggregates the $B^k_{ij\chi}$'s over the two organizational modes.

The above expression for π^k_{ij} should resonate with readers familiar with Eaton and Kortum (2002). This probability of sourcing from country i (regardless of organizational mode) resembles that in the baseline Eaton-Kortum model, except that each country's technology parameter $T^k_{i'}$ has been replaced with $T^k_{i'}(B^k_{i'j})^{\theta^k}$ in both the numerator and denominator of (19). Each $B^k_{i'j}$ depends on $B^k_{i'jV}$ and $B^k_{i'jO}$, which are in turn deep functions of model parameters as spelled out in equations (13), (14), and (16). While the full algebraic expression for each $B^k_{i'j}$ is complicated, we can nevertheless characterize precisely how these key terms vary with the contractibility of headquarter and supplier tasks:

Lemma 1 *For each source country i, destination country j, and input industry k, B^k_{ijV}, B^k_{ijO} and B^k_{ij} are increasing in both μ^k_{hij} and μ^k_{xij}. Furthermore, B^k_{ijV}, $B^k_{ijO} \le 1$ and $(B^k_{ij})^{\theta^k} \le 2^{1-\lambda_i}$, with equality if and only if $\mu^k_{hij} = 1$ and $\mu^k_{xij} = 1$.*

The proof of this lemma is in Appendix A.1. This result is important because it allows us to interpret the B^k_{ij}'s as terms that reflect *contracting capacity*: When multiplied against T^k_i, $(B^k_{ij})^{\theta^k}$ is a wedge that encapsulates how the incomplete contracting environment de facto diminishes the technology for industry-k inputs that is accessible to country-j firms when they source these inputs from country i. Improvements in contractibility – i.e., increases in either μ^k_{hij} or μ^k_{xij} – close the gap between the full technology, $2^{1-\lambda_i}T^k_i$, and that de facto available to country-j firms, $(B^k_{ij})^{\theta^k} T^k_i$; only in the limit where $\mu^k_{hij} = \mu^k_{xij} = 1$ and contracting capacity is complete, is the full technology accessible. The structure of our model thus summarizes the impact of contracting frictions on sourcing patterns through these $(B^k_{ij})^{\theta^k}$ terms, in a manner that resembles conventional iceberg trade costs, $(d^k_{ij})^{-\theta^k}$.

It is instructive to look at several special cases to reinforce intuition. When $\lambda_i = 1$, the $\chi = V$ and $\chi = O$ draws that emanate from country i are perfectly correlated; in this case, $(B^k_{ij})^{\theta^k} \leq 2^{1-\lambda_i} = 1$, which aligns with a situation where each country-j firm has in effect a single draw from a technology that is at best equal to T^k_i. On the other hand, when $\lambda_i = 0$, we have $(B^k_{ij})^{\theta^k} \leq 2$; so with completely uncorrelated draws, each firm de facto has two independent draws from a Fréchet distribution with a scale parameter that is at best equal to T^k_i.

Equation (19) for the π^k_{ij}'s has several familiar implications. The relative probability that a firm sources from a country-i supplier as opposed to one from country i' is:

$$\frac{T^k_i}{T^k_{i'}} \frac{(d^k_{ij} w_i)^{-\theta^k} (B^k_{ij})^{\theta^k}}{(d^k_{i'j} w_{i'})^{-\theta^k} (B^k_{i'j})^{\theta^k}}.$$

A country-j firm will therefore be more likely to source from country i if the state of technology there, T^k_i, is high relative to that in i', $T^k_{i'}$. Sourcing is also more likely to take place with country i if labor costs are lower, shipping to country j is less costly, or if contracting capacity is more complete, relative to i'.

Turning to $\pi^k_{\chi|ij}$, (19) implies that conditional on sourcing from country i, the probability that this occurs under integration can be written as:

$$\pi^k_{V|ij} = \frac{(B^k_{ijV}/B^k_{ijO})^{\frac{\theta^k}{1-\lambda_i}}}{(B^k_{ijV}/B^k_{ijO})^{\frac{\theta^k}{1-\lambda_i}} + 1}. \tag{20}$$

This is an expression for the *intrafirm trade share*, or more literally, the share of input varieties procured under integration. Based on Lemma 1, the B^k_{ijV} and B^k_{ijO} terms that appear in (20) capture contracting capacity when sourcing under the respective organizational modes; bear in mind from (16) that the $B^k_{ij\chi}$'s, and hence the intrafirm trade share, are ultimately functions of the contractibility and bargaining parameters (μ^k_{hij}, μ^k_{xij}, β^k_{ijV}, β^k_{ijO}), as well as other production parameters (θ^k, λ_i, ρ^k, α^k).

In Lemma 2 below, we use (20) to establish how the prevalence of integration in the firm's sourcing strategy is affected by key parameters. In anticipation of how we will implement the model quantitatively, we work with a specification from Antràs (2003) that relates the bargaining share under integration (β^k_{ijV}) to that under outsourcing (β^k_{ijO}). Building off the notion that ownership confers residual rights of control, we assume that under integration, the firm is assured a fraction $\delta^k_{ij} \in (0, 1)$ of the ex-post surplus should the relationship with the supplier break down (although this will ultimately be a scenario that is off the equilibrium path). The firm and the integrated supplier engage in Nash bargaining over the remaining $1 - \delta^k_{ij}$ share of the surplus, splitting this in the ratio β^k_{ijO} to $1 - \beta^k_{ijO}$, according to the two parties' primitive bargaining power under arm's-length sourcing. The firm's bargaining share under integration is thus: $\beta^k_{ijV} = \delta^k_{ij} + \beta^k_{ijO}(1 - \delta^k_{ij}) \in (0, 1)$.

We can now state the following comparative static results (see Appendix A.1 for proofs):

Lemma 2 *The probability of intrafirm sourcing conditional on sourcing from country i, $\pi^k_{V|ij}$, is: (i) increasing in α^k; (ii) increasing in μ^k_{hij}, but decreasing in μ^k_{xij}; (iii) decreasing in β^k_{ijO}; and (iv) decreasing in δ^k_{ij} over the entire interval $\delta^k_{ij} \in (0, 1)$ when α^k is below a cutoff level $\bar{\alpha}^k$, while exhibiting an inverted U-shape relationship with δ^k_{ij} when $\alpha^k > \bar{\alpha}^k$.*

Despite its involved structure, the model delivers clear predictions on the forces that shape the propensity to integrate ($\pi^k_{V|ij}$). The result in (i) is the familiar Grossman-Hart-Moore insight: With a higher α^k, headquarter services become relatively more important vis-à-vis supplier inputs. It is then more crucial to incentivize the firm to invest in its noncontractible tasks, which is better achieved with the favorable division of surplus it would receive under integration ($\beta^k_{ijV} > \beta^k_{ijO}$). The rationale for (ii) likewise works off how the organizational mode affects each party's incentives to invest in their noncontractible effort. A higher μ^k_{xij} allows the firm to spell out the investment levels for a larger subset of supplier tasks. This reduces the need to induce supplier effort, prompting the firm to instead prioritize retaining a larger share of the ex-post surplus and thereby raising the propensity to integrate. Conversely, when headquarter tasks are more contractible (μ^k_{hij} is higher), this heightens the need to incentivize suppliers relative to firm headquarters; the firm would then rather commit to procure more input varieties through outsourcing (c.f., Antràs and Helpman, 2008).

Turning to (iii), an increase in β^k_{ijO} raises the firm's bargaining share under outsourcing, while closing the gap between this and β^k_{ijV}.[19] This makes outsourcing more attractive relative to integration, so that $\pi^k_{V|ij}$ falls. The result in (iv) is more subtle: An increase in the firm's residual control rights would disincentivize supplier effort if the supplier were integrated by the firm. When α^k is sufficiently small, this is detrimental: it discourages effort from the party (the supplier) responsible for the relatively important input, so the propensity to integrate falls unambiguously as δ^k_{ij} increases in this case. On the other hand, when α^k is large, an increase in δ^k_{ij} from an initially low value would improve the firm's willingness to invest in its noncontractible tasks under integration, and this can be beneficial when headquarter services are important. That said, once δ^k_{ij} is sufficiently large, this curtails the supplier's effort to such an extent that it outweighs the benefit of granting better incentives to the firm; the propensity to integrate thus falls eventually with further increases in δ^k_{ij}.[20]

The parameters θ^k and λ_i, which govern the supplier technology draws, also affect the prevalence of integration. When $B^k_{ijV}/B^k_{ijO} < 1$, equation (20) implies that $\pi^k_{V|ij} < 1/2$; intuitively, the firm would find integration less attractive when it has lower contracting capacity under integration than under outsourcing. A larger θ^k or higher λ_i would then reduce $\pi^k_{V|ij}$ and further dampen the likelihood of integration. The converse is true when $B^k_{ijV}/B^k_{ijO} > 1$, as $\pi^k_{V|ij} > 1/2$ and a larger θ^k or λ_i would now reinforce this propensity to integrate. Put otherwise, with nested Fréchet draws that exhibit a smaller variance, or with more highly correlated draws within country "nests", this accentuates the role of non-stochastic forces – in particular, the ratio of contracting capacities B^k_{ijV}/B^k_{ijO} – in determining the share of inputs that are integrated.

Two remarks are in order before we turn to the next section. First, note that π^k_{ij} in (19) and $\pi^k_{V|ij}$ in (20) are shares based on unweighted counts of input varieties. Unlike Eaton and Kortum (2002), our setup departs from a perfectly competitive benchmark – payoffs are determined via bargaining and firms retain ex-post profits – and so π^k_{ij} in (19), for example, is not equal to the share by *value* of imports from country i. This is slightly inconvenient, as the import and intrafirm shares recorded in standard datasets are based on trade values (rather than counts of product varieties). When we aggregate the model, we therefore also derive expressions for the value of trade by organizational mode, and hence

[19]Specifically, $\beta^k_{ijV} - \beta^k_{ijO} = \delta^k_{ij}(1 - \beta^k_{ijO})$, which decreases as β^k_{ijO} rises, holding δ^k_{ij} constant.

[20]While this potential non-monotonicity is interesting, it turns out that for the range of α^k that is most empirically relevant in our quantification (around 0.3 to 0.4), this is low enough that $\pi^k_{V|ij}$ is in practice decreasing in δ^k_{ij} over the whole unit interval (unless some of the other parameters take on extreme values).

for the intrafirm trade share by value, that we can map to the data. As it turns out, the intrafirm trade share by value inherits comparative static properties similar to those for $\pi^k_{V|ij}$ in Lemma 2. This well-behaved nature with respect to the model's primitives gives us a measure of confidence that the model parameters can ultimately be estimated from data on the intrafirm trade share by value.

Second, it is worth discussing how one can incorporate firm-level heterogeneity in sourcing decisions into this framework. A natural way to proceed would be to introduce fixed costs of activating individual source countries and/or organizational modes (c.f., Antràs et al., 2017). Differences in sourcing shares would then emerge, as firms with higher core productivity ϕ would be able to incur the fixed costs of opening up more sourcing modes. In the current paper, we have eschewed going in this direction, since we lack firm-level data on global sourcing patterns broken down for detailed inputs by organizational mode. With such fixed costs, we would moreover lose the exact aggregation of the model with analytically tractable expressions. We have thus opted to leave this as an extension for future work. Our approach here will instead be to work with the closed-forms for trade flows and the intrafirm trade share, to take these directly to the data; note too that at the level of aggregation – NAICS 3-digit industries – that we will adopt, there are relatively few zero trade observations (see Section 4 for details).

3 Aggregate Implications: Welfare and Trade Flows

We now aggregate over the decisions made by the final-good producers in a given country, to derive implications for welfare and for bilateral trade flows. We outline the main steps and highlight key forces in this section; detailed derivations are in Appendix A.2.

3.1 Output, Consumption, and Welfare

Output and consumption: We solve out the firm decision problem in full, to obtain the quantity of each final-good variety that will enter the utility function in (1). For each country-j firm, this entails that we derive expressions for the composite input $X^k_j(\phi)$ it uses from each industry k, and for the assembly labor $L_j(\phi)$ it employs.

From (3), we have: $X^k_j(\phi) = (\mathbb{E}[\tilde{x}^k_j(l^k;\phi)^{\rho^k}])^{\frac{1}{\rho^k}}$, where this expectation is evaluated over the distribution of supplier draws associated with the sourcing modes that are ultimately chosen for input varieties $l^k \in [0,1]$. Making use of the optimal task levels in (12), one can show that:

$$X^k_j(\phi) = (1-\alpha)\rho\eta^k R_j(\phi)(\alpha^k/s_j)^{\alpha^k}(1-\alpha^k)^{1-\alpha^k}(\bar{\Gamma}^k)^{\frac{1-\rho^k}{\rho^k}}(\Phi^k_j)^{\frac{1-\alpha^k}{\theta^k}}(\Upsilon^k_j)^{\frac{1-\rho^k}{\rho^k}}, \tag{21}$$

where:

$$\Phi^k_j = \sum_{i'=1}^{J} T^k_{i'}(d^k_{i'j}w_{i'})^{-\theta^k}(B^k_{i'j})^{\theta^k}, \tag{22}$$

$$\Upsilon^k_j = \sum_{i=1}^{J}\sum_{\chi\in\{V,O\}} \frac{\zeta^k_{ij}}{\zeta^k_{ij\chi}}\pi^k_{ij}\pi^k_{\chi|ij}, \text{ and} \tag{23}$$

$\bar{\Gamma}^k$ is a constant equal to the Gamma function evaluated at $1 - \frac{1}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k} > 0$; this inequality is a restriction on θ^k, ρ^k, and α^k needed for $\bar{\Gamma}^k$, and hence firm output, to be well-defined.[21]

[21] Recall that the Gamma function is given by: $\Gamma(t) = \int_0^\infty y^{t-1}e^{-y}dy$ for $t > 0$. The condition $1 - \frac{1}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k} > 0$

Based on (21), the firm's industry-k input use scales up linearly with its revenue, $R_j(\phi)$. There are moreover two key multiplicative terms in (21) – that depend on Φ_j^k and Υ_j^k – whose role we can interpret. First, notice that Φ_j^k in (22) is precisely the denominator of the country-i sourcing share π_{ij}^k in (19). Following Antràs et al. (2017), we refer to Φ_j^k as the *sourcing capability* of country-j firms in procuring industry-k inputs, since Φ_j^k summarizes the available state of technology across all potential source countries after accounting for prevailing labor costs, trade costs, and contracting capacities. From (22), Φ_j^k is clearly increasing in B_{ij}^k, and hence in each of the μ_{hij}^k and μ_{xij}^k parameters (c.f., Lemma 1). In other words, more severe contracting frictions reduce the sourcing capability of the firm and in turn its use of industry-k inputs embodied in $X_j^k(\phi)$.

Second, Υ_j^k in (23) is also affected by the contractibility parameters. Recall that $\zeta_{ij}^k/\zeta_{ij\chi}^k \leq 1$, so that (23) implies: $\Upsilon_j^k \leq \sum_{i=1}^J \sum_{\chi\in\{V,O\}} \pi_{ij}^k \pi_{\chi|ij}^k = 1$. The $(\Upsilon_j^k)^{(1-\rho^k)/\rho^k}$ term in (21) thus acts like a friction that impairs $X_j^k(\phi)$ from the maximum value it would otherwise attain; $\zeta_{ij}^k/\zeta_{ij\chi}^k$ is moreover increasing in μ_{hij}^k and μ_{xij}^k, so Υ_j^k reaches its maximum value of 1 only when $\mu_{hij}^k = \mu_{xij}^k = 1$ for all source countries i.[22] For this reason, we refer to Υ_j^k as an *under-investment friction* term: Υ_j^k captures the further effect of contracting frictions in dampening investment in headquarter and supplier tasks, after conditioning on the level of the firm's sourcing capability Φ_j^k.

Turning to $L_j(\phi)$, the firm chooses this to maximize its overall payoff (inclusive of the ex-ante transfer). This payoff, which we denote by $F_j(\phi)$, is the revenue from the final good, $R_j(\phi)$, less all factor costs incurred; the latter comprises the payments for headquarter and supplier tasks across all input varieties l^k, as well as for assembly labor. Note that we have specified that both supplier tasks and assembly be performed by the same factor of production (labor); this is not crucial in the theory, but will help us to better match the labor share in aggregate income in our quantitative work.[23] With some algebra, $F_j(\phi)$ can be written as: $F_j(\phi) = \varpi_j R_j(\phi) - w_j L_j(\phi)$, where:

$$\varpi_j = 1 - (1-\alpha)\sum_{k=1}^K \frac{\rho\eta^k}{\rho^k}\left(1 - (1-\rho^k)(\Upsilon_j^k)^{-1}\right). \tag{24}$$

In Appendix A.2, we show that $0 < \varpi_j < 1$.[24] This allows us to interpret ϖ_j as the firm's (pre-assembly) *profit share*, or equivalently, the share of $R_j(\phi)$ that accrues to the firm after netting out the costs of headquarter and supplier tasks. The first-order condition of $F_j(\phi)$ with respect to $L_j(\phi)$ then yields:

$$L_j(\phi) = \frac{\alpha\rho}{1-\rho(1-\alpha)}\frac{\varpi_j}{w_j}R_j(\phi). \tag{25}$$

The assembly labor employed is therefore also a linear function of firm revenue. Note that a higher profit share, ϖ_j, naturally induces the firm to raise its output and hence also $L_j(\phi)$. With the expression for $L_j(\phi)$ from (25), one can further show that the firm's payoff simplifies to: $F_j(\phi) = \frac{1-\rho}{1-\rho(1-\alpha)}\varpi_j R_j(\phi)$; the firm's post-assembly profit share – after accounting for assembly wages – is thus just ϖ_j scaled by a multiplicative constant.

ensures that the dispersion of the nested Fréchet draws is not too large; without this, the power function of these draws that is relevant for evaluating $X_j^k(\phi)$ would not have a finite expectation.

[22] That $\zeta_{ij}^k/\zeta_{ij\chi}^k$ is increasing in μ_{hij}^k and μ_{xij}^k follows from a quick application of the quotient rule; substituting in $\mu_{hij}^k = \mu_{xij}^k = 1$ shows that the maximum value of $\zeta_{ij}^k/\zeta_{ij\chi}^k$, and hence of Υ_j^k, is 1.

[23] To be more specific, these factor payments are: (i) $s_j\int_0^1 h_j^k(\iota_h;\phi,l^k)d\iota_h$ for the headquarter tasks, for each input variety l^k; (ii) $c_{ij\chi}^k(l^k;\phi)\int_0^1 x_j^k(\iota_x;\phi,l^k)d\iota_x$ for the supplier tasks, for each l^k; and (iii) $w_jL_j(\phi)$ for final assembly labor.

[24] This property of ϖ_j follows from showing that the under-investment friction terms satisfy: $1 \geq \Upsilon_j^k \geq 1-\rho^k$.

Combining the expressions for composite inputs from (21) and assembly labor from (25) into the production function in (2), the quantity of each final-good variety is then given by:

$$q_j(\phi) = A_j(\phi\rho)^{\frac{1}{1-\rho}}(1-\alpha)^{\frac{1-\alpha}{1-\rho}}\left(\frac{\alpha}{1-\rho(1-\alpha)}\frac{\varpi_j}{w_j}\right)^{\frac{\alpha}{1-\rho}}$$
$$\times\prod_{k=1}^{K}\left[\left(\frac{\alpha^k}{s_j}\right)^{\alpha^k}(1-\alpha^k)^{1-\alpha^k}\eta^k(\bar{\Gamma}^k)^{\frac{1-\rho^k}{\rho^k}}(\Phi_j^k)^{\frac{1-\alpha^k}{\theta^k}}(\Upsilon_j^k)^{\frac{1-\rho^k}{\rho^k}}\right]^{\eta^k\frac{1-\alpha}{1-\rho}}, \tag{26}$$

where we have made use of: $R_j(\phi) = A_j^{1-\rho}q_j(\phi)^\rho$ in deriving the above.

Welfare: Utility from (1) can be written as: $U_j = (N_j\int_\phi q_j(\phi)^\rho dG_j(\phi))^{\frac{1}{\rho}}$, into which we now substitute $q_j(\phi)$ from (26). This delivers the following expression for country welfare:

$$U_j = (N_j)^{\frac{1-\rho}{\rho}}\rho E_j\bar{\phi}_j(1-\alpha)^{1-\alpha}\left(\frac{\alpha}{1-\rho(1-\alpha)}\frac{\varpi_j}{w_j}\right)^{\alpha}$$
$$\times\prod_{k=1}^{K}\left[\left(\frac{\alpha^k}{s_j}\right)^{\alpha^k}\left(\frac{1-\alpha^k}{w_j}\right)^{1-\alpha^k}\eta^k(\bar{\Gamma}^k)^{\frac{1-\rho^k}{\rho^k}}(\Upsilon_j^k)^{\frac{1-\rho^k}{\rho^k}}\left(\frac{T_j^k}{\pi_{jj}^k}\right)^{\frac{1-\alpha^k}{\theta^k}}(B_{jj}^k)^{1-\alpha^k}\right]^{\eta^k(1-\alpha)}, \tag{27}$$

where: $\bar{\phi}_j \equiv (\int_\phi \phi^{\frac{\rho}{1-\rho}}dG_j(\phi))^{\frac{1-\rho}{\rho}}$ is an aggregator of firms' core productivity levels. To arrive at (27), we have performed the substitution, $\Phi_j^k = T_j^k(w_j)^{-\theta^k}(B_{jj}^k)^{\theta^k}/\pi_{jj}^k$, that comes from setting $i=j$ in (19) while bearing in mind that $d_{jj}^k = 1$. The sourcing capability as defined in (22) depends on prevailing technologies, trade costs, wages, and contracting capabilities, $T_i^k(d_{ij}^k w_i)^{-\theta^k}(B_{ij}^k)^{\theta^k}$, across all countries i in the world; conveniently though, Φ_j^k can be expressed in terms of just the country-j analogue of these variables, $T_j^k(w_j)^{-\theta^k}(B_{jj}^k)^{\theta^k}$, suitably re-scaled by the share of domestic inputs, π_{jj}^k.

For our welfare counterfactuals, we adopt the "hat algebra" approach (c.f., Dekle et al., 2008) and write down (27) in proportional changes. For any variable X, let X' denote its value following a given shock and $\hat{X} \equiv X'/X$ the associated proportional change. The response of country welfare to a shock to either: (i) trade costs (the d_{ij}^k's); or to (ii) contracting conditions (via the μ_{hij}^k's or μ_{xij}^k's), is:

$$\widehat{U_j} = \widehat{E_j}\,(\widehat{w_j})^{-\alpha}\left(\prod_{k=1}^{K}\left[(\widehat{w_j})^{-(1-\alpha^k)}\,(\widehat{s_j})^{-\alpha^k}\right]^{\eta^k(1-\alpha)}\right)(\widehat{\varpi_j})^{\alpha}\prod_{k=1}^{K}\left[\left(\widehat{\Upsilon_j^k}\right)^{\frac{1-\rho^k}{\rho^k}}\left(\widehat{\pi_{jj}^k}\right)^{-\frac{1-\alpha^k}{\theta^k}}(\widehat{B_{jj}^k})^{1-\alpha^k}\right]^{\eta^k(1-\alpha)}. \tag{28}$$

Equation (28) shares key features with the gains-from-trade formula from Arkolakis et al. (2012). At its core, the change in welfare is inversely related to changes in the domestic-sourcing share, $\widehat{\pi_{jj}^k}$, in each industry, and the strength of this dependence on $\widehat{\pi_{jj}^k}$ is governed (in part) by the distance elasticity θ^k. While this term looks familiar, it bears reminding that in our setting, π_{jj}^k is strictly the domestic share by (unweighted) counts of input varieties; we will later establish a model-consistent mapping between this and the sourcing share by value when we go to the data in Section 5.

The welfare change expression is augmented by several terms related to contracting frictions. First, $\widehat{B_{jj}^k}$ appears in (28) due to the earlier substitution made for the sourcing capability, $\Phi_j^k = T_j^k(w_j)^{-\theta^k}(B_{jj}^k)^{\theta^k}/\pi_{jj}^k$. Thus, $\widehat{B_{jj}^k}$ and $\widehat{\pi_{jj}^k}$ jointly speak to the impact that an improvement in contracting conditions would have in raising the sourcing capability of country-j firms over industry-k inputs.

In the scenarios we consider, we will see that $\widehat{B^k_{jj}}$ is in practice an order of magnitude larger than $\widehat{\pi^k_{jj}}$, so the impact of contracting frictions on sourcing capability operates almost entirely via $\widehat{B^k_{jj}}$. Next, the $\widehat{\Upsilon^k_j}$ term in (28) reflects how a shock to the contracting environment would affect the effort invested in headquarter and supplier tasks, over and above its effect on sourcing capability. This follows from our interpretation of Υ^k_j as an under-investment friction: An improvement in contractibility – in μ^k_{hij} or μ^k_{xij} for any source country i – would alleviate the holdup problem between the firm and its industry-k suppliers; this would raise Υ^k_j and, in turn, country-j welfare.

At the same time, there are countervailing forces on welfare. The $\widehat{\varpi_j}$ term in (28) reflects the change in the share of revenues retained by firms as profits. From (24), an improvement in contractibility that raises Υ^k_j also increases the share of $R_j(\phi)$ that is paid out to factors of production; as the profit share ϖ_j then falls, this tends to dampen firm output, and hence the welfare response to an improvement in contracting conditions. Last but not least, (28) contains terms in $\widehat{w_j}$, $\widehat{s_j}$ and $\widehat{E_j}$, which are the endogenous movements in factor prices and aggregate expenditure. The magnitudes of these responses will be pinned down by factor market-clearing conditions in general equilibrium (see Section 3.3 below). But there is clearly scope, for example, for an increased demand for factors to drive up w_j and/or s_j, which could offset some of the consumption gains when contracting frictions are removed.[25]

The presence of these rich forces in (28) points to the need for a quantitative assessment, in order to size up the overall welfare impact of contracting frictions in global sourcing. Before proceeding further, there is a special case to highlight: In a world with only one factor of production (i.e., headquarter services, supplier tasks, and assembly all use a single form of labor), and under full contractibility ($\mu^k_{hij} = \mu^k_{xij} = 1$ for all i, j, and k), the welfare change formula simplifies to:

$$\widehat{U_j} = \prod_{k=1}^{K} (\widehat{\pi^k_{jj}})^{-\frac{\eta^k(1-\alpha^k)}{\theta^k}(1-\alpha)}. \tag{29}$$

Our model with contracting frictions thus nests the gains-from-trade formula for the class of multi-industry models considered in Arkolakis et al. (2012) and Costinot and Rodríguez-Clare (2014).[26]

3.2 Trade flows

For trade flows, we take the position that customs data record the value of shipped items at cost; this is in line with our modeling assumption that a large pool of potential suppliers in each country competes away any supplier rents. With this convention, we compute in Appendix A.2 the value of industry-k inputs that a country-j firm with core productivity ϕ would import under sourcing mode (i, χ), and then aggregate this over the distribution $G_j(\phi)$. The resulting expression for trade flows, $t^k_{ij\chi}$, is:

$$t^k_{ij\chi} = a^k_{ij} a^k_{ij\chi}, \tag{30}$$

where:

$$\left.\begin{aligned} a^k_{ij} &= (1-\alpha)\rho\eta^k(1-\alpha^k)\frac{E_j}{\Upsilon^k_j \Phi^k_j} T^k_i (w_i)^{-\theta^k} (B^k_{ij})^{-\frac{\theta^k \lambda_i}{1-\lambda_i}} (d^k_{ij})^{-\theta^k}, \text{ and} \\ a^k_{ij\chi} &= (B^k_{ij\chi})^{\frac{\theta^k}{1-\lambda_i}} \left(\mu^k_{xij} + (1-\mu^k_{xij})(1-\beta^k_{ij\chi})\frac{\zeta^k_{ij}}{\zeta^k_{ij\chi}} \right). \end{aligned}\right\} \tag{31}$$

[25]The proportional change in w_j appears twice in (28) because labor is used both in supplier tasks and in assembly. If one were to designate assembly labor as a third factor of production, then one would simply replace the $(\widehat{w_j})^{-\alpha}$ term in (28) by the proportional change in that distinct factor raised to the power of $-\alpha$.

[26]This follows from the fact that in the full contractibility world, $B^k_{jj} = 1$, $\Upsilon^k_j = 1$, and $\varpi_j = 1 - \rho(1-\alpha)$.

Note that a_{ij}^k collects: (i) terms specific to the importing country j and industry k, including aggregate expenditure (E_j) and sourcing capabilities (Φ_j^k); (ii) terms specific to the exporting country i and industry k, including variables related to supplier technology (T_i^k) and factor costs (w_i); together with (iii) trade costs and contracting capacities – d_{ij}^k and B_{ij}^k – that vary by country pair and industry. Based on this, (30) can be viewed as a gravity equation for bilateral trade flows by industry and sourcing mode, with a trade cost elasticity of θ^k.

In turn, $a_{ij\chi}^k$ comprises: (iv) terms that depend further on the organizational mode χ, that involve (among other parameters) the contractibility of headquarter and supplier tasks, as well as the bargaining shares. These will feature prominently as drivers of the intrafirm trade share, namely the share by value of trade flows that are brought in under integration rather than outsourcing, $t_{ijV}^k/(t_{ijV}^k + t_{ijO}^k)$.

3.3 Closing the Model

We briefly describe how we close the model, by pinning down w_j, s_j, and E_j in general equilibrium via domestic factor market-clearing conditions (see Appendix A.2 for the detailed derivations).

We denote the given endowments of labor and capital in country j by $\bar{L}_j$ and $\bar{K}_j$ respectively. Equating $\bar{L}_j$ with the various sources of demand for labor yields:

$$w_j \bar{L}_j = \frac{\alpha\rho\varpi_j}{1-\rho(1-\alpha)} E_j + (1-\alpha)\rho \sum_{k=1}^{K}\sum_{m=1}^{J}\sum_{\chi\in\{V,O\}} \eta^k (1-\alpha^k) \frac{E_m}{\Upsilon_m^k} \pi_{jm}^k \pi_{\chi|jm}^k \left(\mu_{xjm}^k + (1-\mu_{xjm}^k)(1-\beta_{jm\chi}^k)\frac{\zeta_{jm}^k}{\zeta_{jm\chi}^k}\right). \tag{32}$$

The first term on the right-hand side arises from payments to assembly labor by country-j firms; the second term in turn accounts for the labor used by country-j suppliers in supplier tasks, for input varieties that are sourced by firms around the world (including by country-j's own final-good producers) under both possible organizational modes.

As for capital in country j, the corresponding factor market-clearing condition is:

$$s_j \bar{K}_j = (1-\alpha)\rho \sum_{k=1}^{K}\sum_{i=1}^{J}\sum_{\chi\in\{V,O\}} \eta^k \alpha^k \frac{E_j}{\Upsilon_j^k} \pi_{ij}^k \pi_{\chi|ij}^k \left(\mu_{hij}^k + (1-\mu_{hij}^k)\beta_{ij\chi}^k \frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k}\right), \tag{33}$$

where note that capital is used only to perform headquarter services for input varieties.

To fully close the model, we assume that the profits of firms are rebated to consumers via a domestic asset market and that these assets are not traded across borders; following Dekle et al. (2008), we treat trade deficits, D_j, as exogenous and given in the data. Aggregate expenditure in country j is then equal to the sum of payments to the two factors ($w_j\bar{L}_j$, $s_j\bar{K}_j$), profits of domestic final-good firms, and the trade deficit. We show in Appendix A.2 this implies the following expression for E_j:

$$E_j = \frac{w_j\bar{L}_j + s_j\bar{K}_j + D_j}{1 - \frac{1-\rho}{1-\rho(1-\alpha)}\varpi_j}. \tag{34}$$

We round off our model description by laying out the system of equations that define an equilibrium in this world of global sourcing with contracting frictions. In words, this comprises: the sourcing shares, π_{ij}^k and $\pi_{\chi|ij}^k$; the definitions of ζ_{ij}^k and $\zeta_{ij\chi}^k$; the contracting capacities, B_{ijV}^k, B_{ijO}^k, and B_{ij}^k; sourcing capabilities, Φ_j^k; under-investment frictions, Υ_j^k; aggregate profit shares, ϖ_j; as well as the labor and capital market-clearing conditions and aggregate expenditure condition (just presented above). For completeness, we list this equilibrium system in equations (A.26)-(A.37) in Appendix A.3.

4 Taking the Model to Intrafirm Trade Data

We now take the trade flow expressions by sourcing mode to the data on intrafirm versus arm's-length trade, from which we will estimate the key parameters of the model.

4.1 Estimation Strategy

Intrafirm trade share (by value): To map (30) to an empirical specification, we assume that trade flows in the data, denoted by $\tilde{t}^k_{ij\chi}$, are equal to their model-based counterparts but observed with noise: $\tilde{t}^k_{ij\chi} = t^k_{ij\chi}\epsilon^k_{ij\chi} = a^k_{ij}a^k_{ij\chi}\epsilon^k_{ij\chi}$, where the multiplicative error term, $\epsilon^k_{ij\chi}$, satisfies $\mathbb{E}[\epsilon^k_{ij\chi}|a^k_{ij\chi}, a^k_{ij}] = 1$.

Drawing on the gravity equation literature, it is then natural to take a Poisson Pseudo-Maximum Likelihood (PML) approach in which we account for the a^k_{ij}'s with country-pair-by-industry fixed effects (Santos Silva and Tenreyro, 2006). In Appendix B.1, we show that when the a^k_{ij}'s are replaced by their corresponding Poisson PML estimators, this delivers a moment condition for the intrafirm trade share by value:

$$E\left[\frac{\tilde{t}^k_{ijV}}{\tilde{t}^k_{ijV}+\tilde{t}^k_{ijO}}\middle|a^k_{ij\chi}, a^k_{ij}\right] = \frac{a^k_{ij}a^k_{ijV}}{a^k_{ij}a^k_{ijV}+a^k_{ij}a^k_{ijO}} = \frac{a^k_{ijV}}{a^k_{ijV}+a^k_{ijO}} = \frac{(B^k_{ijV}/B^k_{ijO})^{\frac{\theta^k}{1-\lambda_i}}(b^k_{ijV}/b^k_{ijO})}{(B^k_{ijV}/B^k_{ijO})^{\frac{\theta^k}{1-\lambda_i}}(b^k_{ijV}/b^k_{ijO})+1}, \quad (35)$$

where: $b^k_{ij\chi} \equiv \mu^k_{xij} + (1-\mu^k_{xij})(1-\beta^k_{ij\chi})\frac{\zeta^k_{ij}}{\zeta^k_{ij\chi}}$ for $\chi \in \{V, O\}$; this last step follows from substituting for the $a^k_{ij\chi}$'s based on (31). Note that $\tilde{t}^k_{ijV}/(\tilde{t}^k_{ijV}+\tilde{t}^k_{ijO})$ on the left-hand side will come directly from the data, while the right-hand side is a model-based structural expression for this intrafirm trade share.[27]

Comparing (35) with (20), the ratio b^k_{ijV}/b^k_{ijO} acts as an adjustment term that maps the intrafirm trade share by counts of input varieties, $\pi^k_{V|ij}$ from (20), to an intrafirm trade share based on the value of trade flows.[28] We show in Appendix B.2 that this b^k_{ijV}/b^k_{ijO} term typically reinforces how B^k_{ijV}/B^k_{ijO} – and hence, $\pi^k_{V|ij}$ – responds to each of the underlying model parameters α^k, μ^k_{xij}, μ^k_{hij}, β^k_{ijO}, and δ^k_{ij}. While we clarify in Appendix B.2 that there are parameter values under which some of the comparative static results in Lemma 2 can be overturned, these are extreme cases that are not empirically relevant; it is straightforward to provide a sufficient condition, readily satisfied in practice, under which such scenarios are ruled out.[29] For all practical purposes then, the intrafirm trade share by value in (35) inherits the comparative static properties of $\pi^k_{V|ij}$ laid out in Lemma 2.

There are two advantages to working with (35) as our estimating equation. First, empirical studies on the determinants of organizational mode in global sourcing have, starting with Antràs (2003), regularly adopted this intrafirm trade share (by value) as a key dependent variable to capture the propensity

[27]In Appendix B.1, we also provide an alternative foundation for (35): Under the further assumption that the trade flows, $\tilde{t}^k_{ij\chi}$, are realizations from a Poisson distribution with mean $a^k_{ij}a^k_{ij\chi}$, one can show that conditional on observed total trade $\tilde{t}^k_{ij}$ within a country-pair-by-industry bin, $\tilde{t}^k_{ijV}/\tilde{t}^k_{ij}$ obeys a Bernoulli distribution with mean $a^k_{ijV}/(a^k_{ijV}+a^k_{ijO})$. The derivation is akin to that in Eaton et al. (2013), where the moment condition is not exact but rather holds in expectation after accounting for a multiplicative error term associated with observed trade flows.

[28]One can thus interpret b^k_{ijV}/b^k_{ijO} as a term that governs the intensive margin of intrafirm sourcing. From (20), on the other hand, B^k_{ijV}/B^k_{ijO} governs the probability – and hence, the extensive margin – of intrafirm sourcing.

[29]We show in Appendix B.2 that the sufficient condition is needed to unambiguously sign the response of (35) with respect to α^k and β^k_{ijO}, but is not required to characterize the response with respect to μ^k_{xij}, μ^k_{hij}, and δ^k_{ij}. The sufficient condition takes the form that λ_i be large: If the supplier draws under integration and outsourcing are strongly correlated, a higher α^k (respectively, lower β^k_{ijO}) will raise the intrafirm trade share by value, as there is less scope for the supplier draw under outsourcing to be vastly better than that under integration to overturn the baseline comparative static for $\pi^k_{V|ij}$ from Lemma 2. As we report in Section 4.3, the estimates for λ_i we obtain are all close to 1.

to conduct sourcing under integration rather than at arm's length. Equation (35) thus provides a structural rationalization for the regressions, often of a reduced-form nature, that have been run in this body of empirical work cited earlier in the Introduction. Second, with the intrafirm trade flows expressed in shares rather than in levels, one can purge out forces that only vary at the country and/or industry level (the a_{ij}^k's), and focus on forces that are directly relevant for organizational mode decisions (embodied in the $a_{ij\chi}^k$'s). As the derivation in Appendix B.1 makes clear, the a_{ij}^k's are concentrated out from (35) because, with the Poisson PML estimator, the total predicted trade is precisely equal to the observed trade within each country-pair-by-industry bin, i.e., $a_{ij}^k a_{ijV}^k + a_{ij}^k a_{ijO}^k = \tilde{t}_{ijV}^k + \tilde{t}_{ijO}^k$; the Poisson PML estimator is in fact the unique PML estimator that exhibits this "adding-up" property within fixed effect bins (Arvis and Shepherd, 2013; Fally, 2015).[30]

In what follows, all mentions to the intrafirm trade share will refer to the intrafirm trade share by value, as this is the measure we can observe and work with.

Empirical specification: Our goal is to recover the parameters needed to evaluate the welfare change formula in (28). Based on the definition of $a_{ij\chi}^k$ in (31) and the prior expressions for $B_{ij\chi}^k$, $\zeta_{ij\chi}^k$ and ζ_{ij}^j in (16), (13) and (14), these are: α^k, μ_{hij}^k, μ_{xij}^k, β_{ijO}^k, β_{ijV}^k (or equivalently, δ_{ij}^k), θ^k, λ_i, and ρ^k. We describe the approach we take for each of these model parameters.

For α^k, μ_{hij}^k, μ_{xij}^k, β_{ijO}^k, and δ_{ij}^k, we parameterize these as flexible functions – specifically, second-order polynomials – of industry and country characteristics which have been proposed and explored as explanatory variables for the intrafirm trade share in the related empirical literature. Table 1 summarizes the mappings to observables we adopt. We elaborate on these specification choices below, with further details in Appendix B.3 and B.4.

Starting with α^k, we follow Antràs (2003) in using physical capital intensity in the industry, $\log(K/L)^k$, to proxy for the intensity of headquarter services, on the grounds that the input services attributable to plant, property and equipment stem from investment decisions typically undertaken by firm headquarters. There is moreover an extensive body of evidence originating from Antràs (2003) of a robust positive association between physical capital intensity and the intrafirm trade share. We thus specify: $\alpha^k = \ell\left(\mathbf{a}(\log(K/L)^k)\right)$, where $\log(K/L)^k$ is log real capital per worker, and $\mathbf{a}(\cdot)$ is a quadratic function of its argument. We apply a monotonic transformation, the standard logistic function: $\ell(y) = \exp\{y\}/(1+\exp\{y\})$, so that α^k sits in $[0, 1]$. There are then three parameters to estimate here, namely the constant, linear and squared term coefficients of $\log(K/L)^k$ in $\mathbf{a}(\cdot)$.[31]

We turn next to the contractibility parameters, $\mu_{hij}^k, \mu_{xij}^k \in [0, 1]$. While our model is general enough to have these vary by country pair and industry, it is not feasible to accommodate this full flexibility (with J^2K distinct μ_{hij}^k's and μ_{xij}^k's each) in practice: The estimation would be under-identified since the variation in the intrafirm trade data that we work off is cross-sectional in nature. Instead, we reduce the dimensionality of the estimation problem by specifying μ_{hij}^k and μ_{xij}^k to be functions of a parsimonious list of source and destination country as well as industry characteristics. This is analogous

[30] An alternative estimation approach would be to run a Poisson PML regression of $\tilde{t}_{ij\chi}^k$ against country-pair-by-industry fixed effects; the residuals from this would constitute unbiased estimates of the $a_{ij\chi}^k$'s, which can then be fitted to the model-based expression for $a_{ij\chi}^k$ in (31) to recover the underlying parameters. We have opted not to take this route, as the predicted residuals could incorporate more noise resulting in less efficient estimation. Note too that the comparative statics of a_{ijV}^k and a_{ijO}^k with respect to the model parameters are not as cleanly signed, admitting the possibility of non-monotonicities, compared to the corresponding behavior of $a_{ijV}^k/(a_{ijV}^k + a_{ijO}^k)$.

[31] While prior empirical work on the correlates of the intrafirm trade share has not often explored the relevance of a squared term in $\log(K/L)^k$, we have included it here to maintain the same polynomial degree in functional-form flexibility as we allow for with μ_{hij}^k, μ_{xij}^k, β_{ijO}^k, and δ_{ij}^k.

Model Parameters	Functional Forms	Observables
α^k	$\ell\left(\mathbf{a}(\cdot)\right)$	$\log(K/L)^k$
μ^k_{hij}	$\ell\left(\mathbf{h}(\cdot)\right)$	$\text{ROL}_i, \text{ROL}_j, \text{HQContractibility}^k$
μ^k_{xij}	$\ell\left(\mathbf{x}(\cdot)\right)$	$\text{ROL}_i, \text{ROL}_j, \text{SSContractibility}^k$
β^k_{ijO}	$\ell\left(\mathbf{b}(\cdot)\right)$	Markup^k
δ^k_{ij}	$\ell\left(\mathbf{d}(\cdot)\right)$	$\text{ROL}_i, \text{ROL}_j, \text{Specificity}^k, \text{BIT}_{ij}$

Table 1: Mapping from Parameters to Observables

Notes: This table summarizes the empirical specifications we adopt to express the stated model parameters in terms of industry and country characteristics. $\ell(\cdot)$ is the standard logistic function: $\ell(\cdot) = \exp\{\cdot\}/(1 + \exp\{\cdot\})$. $\mathbf{a}(\cdot)$, $\mathbf{h}(\cdot)$, $\mathbf{x}(\cdot)$, $\mathbf{b}(\cdot)$, and $\mathbf{d}(\cdot)$ are second-order polynomial functions of the listed observables, as spelled out in full in Appendix B.3. See the main text and Appendix B.4 for the definitions of the country and industry observables.

to the standard approach in the gravity equation literature for capturing trade costs, where the iceberg friction is projected onto observables such as physical distance, among other variables (cf., Head and Mayer, 2014). Closer to our setting, Boehm (2022) similarly maps model parameters that capture contracting frictions onto institutional variables related to the enforcement of contracts.

Specifically, we express μ^k_{hij} and μ^k_{xij} as (standard logistic transforms of) second-order polynomial functions, $\mathbf{h}(\cdot)$ and $\mathbf{x}(\cdot)$, of: (i) rule of law in the source and destination countries (ROL_i, ROL_j); and (ii) the contractibility of headquarter (respectively, supplier) inputs used in industry k.[32] The rule of law index is a convenient summary measure of the quality of institutions relevant for contract enforcement. The security of contracts in global sourcing could in principle depend on both source and destination institutions; we do not take an *a priori* stance as to which matters more, and will let the variation in the data inform this. The industry characteristics we use in $\mathbf{h}(\cdot)$ and $\mathbf{x}(\cdot)$ build off Nunn (2007). The measure of relationship-specificity devised by Nunn (2007) captures the share of inputs in an industry that likely feature customization (i.e., are classified as differentiated by Rauch, 1999) and are thus more prone to holdup problems.[33] To capture the converse (i.e., the degree of contractibility), we work with the share of relatively standardized inputs – those either exchange-traded or reference-priced – on the basis that goods transacted on open markets are easier to contract upon. We furthermore seek separate contractibility measures for headquarter and supplier inputs, for μ^k_{hij} and μ^k_{xij} respectively. For this, we designate inputs from NAICS 6-digit industries with an above-median capital-labor ratio to be headquarter inputs, while viewing those with a below-median ratio to be supplier inputs; this is in line with the notion that firm headquarters are more likely to be involved in the provision of capital-intensive tasks. We then compute the share (by value) of exchange-traded or reference-priced inputs within each of these subsets to arrive at the "$\text{HQContractibility}^k$" and "$\text{SSContractibility}^k$" measures.[34] Note that $\mathbf{h}(\cdot)$ includes interaction terms between source-country (respectively, destination-country) rule of law and industry $\text{HQContractibility}^k$; an analogous statement applies to $\mathbf{x}(\cdot)$ with $\text{SSContractibility}^k$. We

[32]The $\mathbf{h}(\cdot)$ and $\mathbf{x}(\cdot)$ functions each include a constant; linear and squared terms in each country and industry characteristic; as well as the interactions between each relevant pair of country or industry variables in the respective arguments (see Appendix B.3).

[33]There is ample precedent of work exploring the correlation between the relationship-specificity of inputs and the propensity towards sourcing under integration (see for example Nunn and Trefler, 2008, 2013; Corcos et al., 2013; Defever and Toubal, 2013; Antràs, 2015; Eppinger and Kukharskyy, 2021).

[34]We compute one minus the z^{rs1} measure as defined in Nunn (2007), but separately for the subsets of above- versus below-median capital-intensity inputs. Across NAICS 3-digit industries, the correlation between $\text{HQContractibility}^k$ and $\text{SSContractibility}^k$ is positive but not very high (0.46); their respective correlations with Specificity^k are -0.49 and -0.08.

thus uncover the relevance of contracting conditions for organizational decisions in global sourcing by examining the effect of ROL_i (respectively, ROL_j), while further exploiting differences across industries in their dependence on these institutions; this is in the same spirit as related interaction specifications in Antràs (2015) and Eppinger and Kukharskyy (2021).[35]

We take a similar functional-form approach for β^k_{ijO} and δ^k_{ij}, which govern bargaining outcomes (recall that: $\beta^k_{ijV} = (1 - \delta^k_{ij})\beta^k_O + \delta^k_{ij}$). For β^k_{ijO}, there is less precedent to draw on for guidance on characteristics that speak to the firm's bargaining share under outsourcing. As a baseline, we map β^k_{ijO} to (a standard logistic transform of) a quadratic function $\mathbf{b}(\cdot)$ of the average price markup in an industry ("Markup^k"), since industries with higher markups are typically dominated by fewer incumbent firms, which could confer stronger bargaining power over suppliers.[36] We have also considered an alternative specification in which we estimate a common β_O across all i, j, and k; this yields broadly similar results to our baseline. As we will see, this is because even under the version of β^k_{ijO} parameterized in terms of Markup^k, the estimated bargaining shares exhibit little cross-industry variation.

For δ^k_{ij} – the bargaining share the firm is minimally assured under integration – we set this as (a standard logistic transform of) the second-order polynomial $\mathbf{d}(\cdot)$, whose arguments are: (i) source- and destination-country rule of law (ROL_i, ROL_j); and (ii) a "Specificity^k" measure of the extent to which the industry's output comprises differentiated products (based on Rauch, 1999). In principle, during a supplier dispute, the firm would be more able to exercise its control rights in institutional settings where the rule of law facilitates the recovery of semi-finished goods; it should likewise enjoy a higher δ^k_{ij} if the goods in question have a *lower* specificity and can be readily resold on an open market (c.f., Eppinger and Kukharskyy, 2021). We also include in $\mathbf{d}(\cdot)$: (iii) the level effect of "BIT_{ij}", an indicator for whether the country pair share a Bilateral Investment Treaty (BIT). Such treaties bolster the security of cross-border investments between signatory countries (e.g., Egger and Pfaffermayr, 2004; Büthe and Milner, 2009; Egger and Merlo, 2012), which could encourage sourcing under integration.[37]

As for the remaining model parameters, we directly estimate the Fréchet dispersion, θ^k, for NAICS 3-digit industries, this being the level of industry aggregation in our implementation. For the λ_i's, we partition the countries into three groups – lower-middle, upper-middle, and high income, as classified by the World Bank – and estimate a common correlation parameter within each of these subsets of exporting countries; this keeps tractable the number of distinct λ_i's to be estimated.[38] Last but not least, we pin down the ρ^k's by bringing in information on import demand elasticities from Soderbery (2015). In Appendix B.3, we show that the model-implied counterpart of the import demand elasticity is $1 + \frac{(1-\alpha^k)\rho^k}{1-\rho^k}$; equating this to the corresponding Soderbery (2015) elasticity (denoted by $\tilde{\sigma}^k$) implies: $\rho^k = \frac{\tilde{\sigma}^k - 1}{\tilde{\sigma}^k - \alpha^k}$, which is an expression we plug into $a^k_{ijV}/(a^k_{ijV} + a^k_{ijO})$ for ρ^k wherever the latter appears. With α^k in the denominator of ρ^k taking on the empirical specification spelled out in the first row of Table 1, equation (35) can then be estimated and the value of ρ^k backed out subsequently.

[35] See Chapter 8 of Antràs (2015). This approach draws on a body of work, surveyed in Nunn and Trefler (2014), that has uncovered patterns of comparative advantage in trade flows, through the use of interaction terms between country characteristics and variables that speak to an industry's dependence on these characteristics for production.

[36] We use the average industry-level markups reported in De Loecker et al. (2016); we are not aware (to date) of data on markups for a large enough set of countries that would allow us to incorporate variation along this further dimension.

[37] In line with this, the BIT coefficient is estimated to be positive and significant; that said, our baseline results are largely unaffected if we drop this BIT_{ij} term from the specification of $\mathbf{d}(\cdot)$.

[38] Our sample is based on the US' 50 largest import partners, which does not include any countries classified as low-income (see Section 4.2).

Estimation procedure: We collect the objects to be estimated in the vector Θ. This comprises: (i) the θ^k's for each NAICS 3-digit industry k; (ii) the λ_i correlation parameters for the three country groups (as described above); and (iii) the constant terms and polynomial coefficients of $\mathbf{a}(\cdot)$, $\mathbf{h}(\cdot)$, $\mathbf{x}(\cdot)$, $\mathbf{b}(\cdot)$, and $\mathbf{d}(\cdot)$. Based on (35), we set up the moment condition:

$$m(\Theta) = \mathbb{E}\left[\frac{\tilde{t}^k_{ijV}}{\tilde{t}^k_{ijV} + \tilde{t}^k_{ijO}} - \frac{a^k_{ijV}}{a^k_{ijV} + a^k_{ijO}} \Big| \mathbf{X}^k_{ij}\right] = 0,$$

where $\mathbf{X}^k_{ij}$ is the full set of country and industry observables used in $\mathbf{a}(\cdot)$, $\mathbf{h}(\cdot)$, $\mathbf{x}(\cdot)$, $\mathbf{b}(\cdot)$, and $\mathbf{d}(\cdot)$.

We perform the estimation via weighted non-linear least squares (NLLS), given formally by:

$$\Theta^* = \text{argmin}_{\Theta}\left[(m(\Theta))^T \cdot \Omega \cdot (m(\Theta))\right], \tag{36}$$

with Ω being a diagonal weighting matrix with $\tilde{t}^k_{ij}$ as its entries; this helps to guard against the estimates being driven by observations that account for only low amounts of trade value.[39] We solve the minimization problem in (36) while imposing the constraints: $\theta^k > 1$, $\lambda_i \in (0,1)$, and $\frac{(1-\alpha^k)\rho^k}{\theta^k(1-\rho^k)} < 1$. (Recall from Section 3.1 that this last restriction ensures the dispersion of the nested-Fréchet draws is not too large, so that firm output is bounded.) Computationally, we use a combination of Particle Swarm Optimization and Levenberg-Marquardt algorithms; the minimand in (36) is in practice well-behaved in the relevant parameter space (see Appendix B.5 for implementation details).

We describe the sources of data variation that help identify the parameters in Θ. For the θ^k's and λ_i's, these are informed by the dispersion in $\tilde{t}^k_{ijV}/(\tilde{t}^k_{ijV}+\tilde{t}^k_{ijO})$ seen within industries and within exporting country groups respectively; this follows from our earlier discussion of how the intrafirm trade share responds to shifts in θ^k and λ_i (see Section 2.4). As for the polynomial coefficients in $\mathbf{a}(\cdot)$, $\mathbf{h}(\cdot)$, $\mathbf{x}(\cdot)$, $\mathbf{b}(\cdot)$, and $\mathbf{d}(\cdot)$, these are disciplined intuitively by the observed covariance between $\tilde{t}^k_{ijV}/(\tilde{t}^k_{ijV}+\tilde{t}^k_{ijO})$ and the corresponding country and/or industry variable in $\mathbf{X}^k_{ij}$. To give a concrete example, the covariance between the intrafirm trade share and $\text{ROL}_i \times \text{Specificity}^k$ in the data will be particularly informative of the coefficient of $\text{ROL}_i \times \text{Specificity}^k$ in the $\mathbf{d}(\cdot)$ function for δ^k_{ij}. There are several variables (e.g., ROL_i, ROL_j) in our implementation that enter more than one of the polynomial functions; fully pinning down the coefficients of these variables (as well as the constant terms in each polynomial) will then rely on the best-fit that the functional form achieves to the data. We nevertheless show in Appendix B.5 that each parameter is numerically well-identified, in that the objective function exhibits a distinct local minimum in the neighborhood of each parameter point estimate (see Figures B.1 and B.2).

It bears noting that the above approach to quantification is by no means limited to the specific country and industry variables we have used in $\mathbf{X}^k_{ij}$. The approach is flexible in the sense that one can include additional (or alternative) covariates deemed to be relevant for parameterizing α^k, μ^k_{hij}, μ^k_{xij}, β^k_{ijO}, and δ^k_{ij} in terms of observables, provided there are sufficient degrees of freedom to estimate the associated coefficients. We will later discuss the robustness of our results under some alternative mappings to observables.

4.2 Data and Sample

We describe the key features of the dataset we assemble for the estimation; additional details are documented in Appendix B.4.

[39] Smaller trade flows could also be more exposed to measurement error concerns, as noise in the data that is additive in nature would constitute a larger share of the trade value in proportional terms.

We construct the intrafirm trade share from the US Census Bureau's Related Party Trade Database; this is calculated as the value of trade between parties with an ownership link (our data analogue for $\tilde{t}^k_{ijV}$) divided by the sum of related and non-related party trade ($\tilde{t}^k_{ijV} + \tilde{t}^k_{ijO}$). Most past studies that have used this data to explore organizational modes in sourcing decisions have focused on the US import flows. We utilize here both the import and export data, as the latter provides variation to identify the role of destination-country characteristics for the key contractibility and bargaining parameters in our model. Note that either i = US or j = US for each observation (either the exporting or importing country is the US), although one could readily implement this estimation procedure using data from more countries should comparable data on trade by intrafirm status become available. The intrafirm trade share exhibits persistence over time, so we average $\tilde{t}^k_{ijV}/(\tilde{t}^k_{ijV} + \tilde{t}^k_{ijO})$ over 2001-2005, to use what is in principle a less noisy measure.

For countries, we focus on the US' 50 largest sources for manufacturing industry imports; we however exclude Iraq, Saudi Arabia, Venezuela, and Hong Kong, as these are predominantly oil-exporting nations or trading gateways for other countries. The remaining 46 countries that are in our sample accounted for 95.2% (91.8%) of US manufacturing imports (exports) in 2001-2005. (See Table B.3 in the appendix for the full country list.)

For the industry units in our analysis, we work with the 21 NAICS 3-digit manufacturing industries. This is relatively aggregate, but allows for more consistency with the model in terms of minimizing the presence of cells with zero trade flows. As constituted, we could have a maximum of $21 \times 46 \times 2 = 1,932$ data points (counting both imports and exports). Six of these cells have zero total trade flows (i.e., $\tilde{t}^k_{ijV} + \tilde{t}^k_{ijO} = 0$); since the intrafirm trade share is not well-defined for these cells, we drop them from the estimation. Of the remaining 1,926 observations, only 15 report zero related party trade (i.e., an intrafirm trade share of zero); there are no entries with an intrafirm trade share of 1.

The country and industry variables in our empirical specifications are drawn from commonly-used data sources. The rule of law index is from the World Bank World Governance Indicators. The BIT indicator is from the World Bank's Database of Bilateral Investment Treaties. As for the industry measures, we compute $\ln(K/L)^k$ from the NBER-CES Manufacturing Industry Database. The Specificity^k variable is based on the Rauch (1999) coding for differentiated products; we bridge this over to NAICS industry categories with concordances as described in Appendix B.4. We merge this measure of specificity with information on input usage by industries from the 1997 US Input-Output Tables in order to construct the HQContractibility^k and SSContractibility^k variables, after sorting the input industries by the prior measure of real physical capital per worker.

4.3 Estimation Results

We present the results from running the estimation procedure on this dataset. As we will see, the supplier-draw correlation within country "nests" turns out to be highly relevant. Moreover, we uncover rich relationships between the model parameters and country and industry characteristics. Some of these corroborate well-established patterns (e.g., that physical capital intensity is positively associated with α^k), while others are more novel (e.g., that we find destination-country rule of law to be a strong correlate of both μ^k_{hij} and μ^k_{xij}).

Goodness of fit: Figure 1 illustrates the intrafirm trade shares that are predicted based on our estimates against the shares that we see in the actual data; larger circles in the figure correspond to data points with higher trade values, $\tilde{t}^k_{ijV} + \tilde{t}^k_{ijO}$. The model delivers a good fit: The weighted correlation between the predicted and actual intrafirm trade shares is 0.76 (computed using $\tilde{t}^k_{ijV} + \tilde{t}^k_{ijO}$ as weights). The weighted root mean squared error is equal to 0.169, which is smaller than (being around two-thirds of) the weighted standard deviation of the intrafirm trade shares in the raw data.

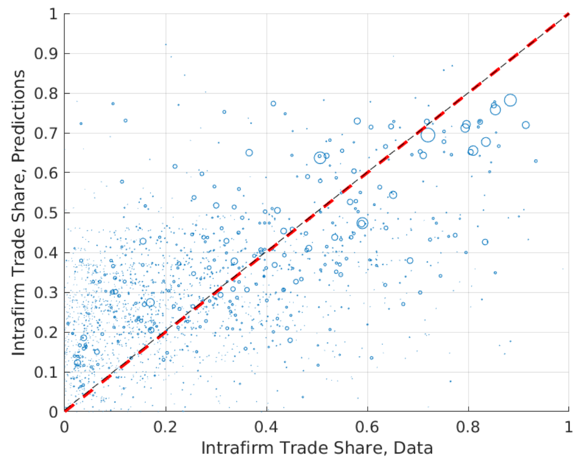


Figure 1: Intrafirm Trade Share, Prediction vs Data

Notes: The figure plots the intrafirm trade shares predicted from our baseline model estimation against that seen in the data. Each point is an observation at the country-pair-by-industry level. The size of the circle is proportional to the weight assigned to the observation, which is the total trade value, $\tilde{t}^k_{ijV} + \tilde{t}^k_{ijO}$. The red dashed line is the 45-degree line.

We perform an additional diagnostic in Appendix B.6 to show how the fitted intrafirm trade shares align well with the actual data. There, we take the predicted intrafirm trade shares (and separately, the raw data) and regress these via OLS directly on $\mathbf{X}^k_{ij}$, the full set of country and industry covariates (including interaction terms) used in our empirical specifications for α^k, μ^k_{hij}, μ^k_{xij}, β^k_O, and δ^k_{ij}; this is in the style of the reduced-form regressions on the determinants of the intrafirm trade share that have been run in prior studies. As Table B.5 shows, the predicted shares correlate with these covariates in a manner that matches the behavior of the actual intrafirm trade shares. The implied magnitudes of the effects of the country and industry variables – when computed either as an average marginal effect or inter-quartile effect – also line up well, even though these are not explicitly targeted as moments by the weighted-NLLS objective function in (36).

Parameter estimates: There are 57 elements in total in Θ, with: 20 industry dispersion parameters (this needs to be normalized for one industry), 3 country-group correlation parameters, and 34 polynomial coefficients.[40] We first describe the estimates for θ^k and λ_i, before discussing what the polynomial coefficient estimates imply for α^k, μ^k_{hij}, μ^k_{xij}, β^k_O, and δ^k_{ij} through a series of figures and surface plots. We refer readers to Table B.4 in Appendix B.6 for the point estimates for all elements of Θ; there, we also report 95% confidence intervals based on 200 bootstrapped samples.[41]

[40] As equations (B.6)-(B.10) in Appendix B.3 make clear, there are three polynomial coefficients each in $\mathbf{a}(\cdot)$ and $\mathbf{b}(\cdot)$, nine each in $\mathbf{h}(\cdot)$ and $\mathbf{x}(\cdot)$, and ten in $\mathbf{d}(\cdot)$. It turns out that the $\text{ROL}_i \times \text{ROL}_j$ term is dropped from the $\mathbf{h}(\cdot)$, $\mathbf{x}(\cdot)$, and $\mathbf{d}(\cdot)$ functions in our implementation, since when the US is either an exporter or importer in each data point, this interaction term can be written as a linear combination of ROL_i, ROL_j and a constant vector (see Appendix B.3 for the derivation); one could reincorporate this interaction term if data on intrafirm trade shares among country pairs that exclude the US were available. Note that $\mathbf{d}(\cdot)$ contains the BIT dummy as an additional covariate.

[41] We report the percentile instead of studentized confidence intervals because the bootstrapped distributions for a

k	NAICS	Description	θ^k	α^k	β_O^k	ρ^k
1	311	Food Manufacturing	4.000	0.357	0.375	0.861
2	312	Beverage and Tobacco Product Manufacturing	16.923	0.401	0.375	0.861
3	313	Textile Mills	4.939	0.366	0.376	0.878
4	314	Textile Product Mills	5.818	0.292	0.376	0.823
5	315	Apparel Manufacturing	6.330	0.282	0.376	0.890
6	316	Leather and Allied Product Manufacturing	8.752	0.301	0.376	0.836
7	321	Wood Product Manufacturing	7.342	0.304	0.374	0.834
8	322	Paper Manufacturing	8.455	0.398	0.377	0.697
9	323	Printing and Related Support Activities	11.804	0.319	0.374	0.764
10	324	Petroleum and Coal Products Manufacturing	9.168	0.420	0.374	0.927
11	325	Chemical Manufacturing	19.860	0.408	0.374	0.850
12	326	Plastics and Rubber Products Manufacturing	6.762	0.343	0.381	0.911
13	327	Nonmetallic Mineral Product Manufacturing	25.483	0.364	0.382	0.816
14	331	Primary Metal Manufacturing	23.719	0.400	0.373	0.931
15	332	Fabricated Metal Product Manufacturing	7.524	0.331	0.376	0.784
16	333	Machinery Manufacturing	13.945	0.352	0.374	0.891
17	334	Computer and Electronic Product Manufacturing	13.572	0.388	0.393	0.814
18	335	Electrical Equipment, Appliance, and Component Manuf ...	1.430	0.343	0.393	0.685
19	336	Transportation Equipment Manufacturing	18.245	0.370	0.382	0.826
20	337	Furniture and Related Product Manufacturing	6.676	0.269	0.374	0.366
21	339	Miscellaneous Manufacturing	5.447	0.312	0.374	0.784
-	-	Mean	0.811	0.349	0.377	10.771

Table 2: Estimation Results, Industry-Level Parameters

Notes: The parameters reported are: θ^k, estimated directly; α^k, as implied by the estimated $\mathbf{a}(\cdot)$ function; β_O^k, as implied by the $\mathbf{b}(\cdot)$ function; and ρ^k, which are backed out using demand elasticities from Soderbery (2015).

As discussed earlier, the θ^k's and λ_i's govern the dispersion in the intrafirm trade shares by industry and within exporting country groups respectively. These parameters can be jointly identified only up to a scaling factor because they enter the nested-Fréchet cdf in (10) via the ratios $\frac{\theta^k}{1-\lambda_i}$. For this reason, we fix θ^1 prior to estimation – specifically, we set $\theta^1 = 4$ for food manufacturing – and pin down all the other θ^k's and λ_i's with this normalization.

The θ^k estimates vary in meaningful ways (see Table 2). Industries with more product differentiation often have lower θ^k values and hence more dispersion in supplier draws; these include "Electrical Equipment" ($\theta^k = 1.43$) and "Apparel" ($\theta^k = 6.33$). On the other hand, industries with more homogeneous goods, such as "Non-Metallic Mineral Products" ($\theta^k = 25.48$) and "Primary Metal Manufacturing" ($\theta^k = 23.72$), feature supplier draws that are more evenly distributed. (All estimated θ^k's are larger than 1 at the 5% significance level based on the bootstrap confidence intervals; see Table B.4.)

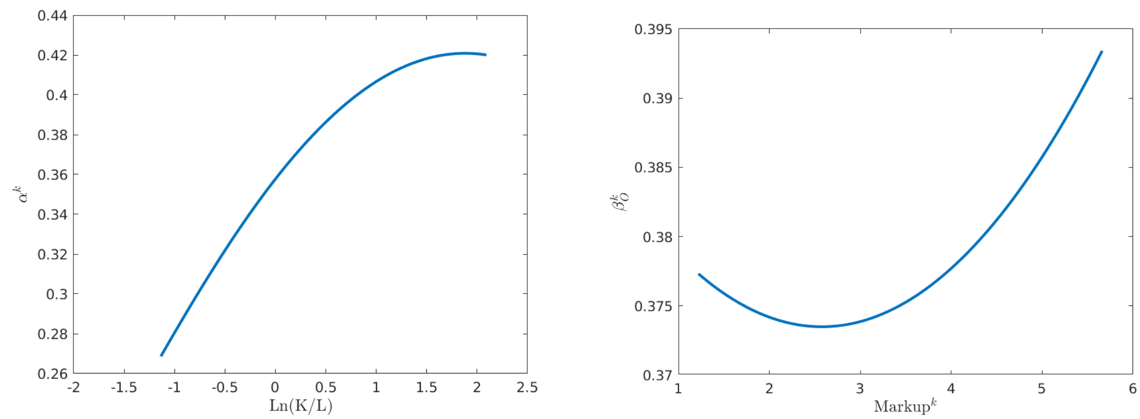
The nested-Fréchet correlation parameters are positive and significant for all three country groups, which underscores the importance for this feature in the distribution of the supplier draws. We obtain estimates of $\lambda_{(1)} = 0.944$ for lower-middle, $\lambda_{(2)} = 0.923$ for upper-middle, and $\lambda_{(3)} = 0.852$ for high income countries, respectively.[42] The supplier draws across organizational modes are thus more dispersed when the source country in question is at a higher level of economic development.

We turn next to discuss the model parameters that are estimated via a mapping to observables. The propensity to integrate is expected to rise with the intensity in use of headquarter inputs (c.f.,

number of parameters exhibit skew (see Figures B.3 and B.4 in Appendix B.6).

[42]The associated bootstraped 95% confidence intervals are $[0.63, 0.99]$, $[0.43, 0.98]$, and $[0.13, 0.96]$, respectively.

Lemma 2), as captured by α^k in the model and proxied by real physical capital per worker in practice (Antràs, 2003). Indeed, the $\mathbf{a}(\cdot)$ function we estimate yields a relationship between α^k and $\ln(K/L)^k$ that is upward-sloping over close to the entire range of the latter variable, as illustrated in Figure 2(a); this is driven by the large significant coefficient we obtain for the linear term in the $\mathbf{a}(\cdot)$ function (see Table B.4). As reported in Table 2, the implied α^k's vary between 0.27 and 0.42.



(a) α^k (b) β^k_O

Figure 2: Estimated α^k and β^k_O

Notes: Panel (a) plots the relationship between $\alpha^k = \ell(\mathbf{a}(\cdot))$ and $\ln(K/L)^k$ implied by our baseline parameter estimates. Panel (b) does analogously for the relationship between $\beta^k_O = \ell(\mathbf{b}(\cdot))$ and Markup^k.

For the contractibility parameters, the $\mathbf{h}(\cdot)$ and $\mathbf{x}(\cdot)$ polynomials allow μ^k_{hij} and μ^k_{xij} to depend flexibly on two country variables (ROL$_i$, ROL$_j$) and one industry characteristic (HQContractibility^k and SSContractibility^k, respectively). Our estimation reveals the key driver of variation in both μ^k_{hij} and μ^k_{xij} to be the rule of law in the destination country. The surface plots in Panels (a)-(d) of Figure 3 show that μ^k_{hij} and μ^k_{xij} are rising in ROL$_j$, due to the positive significant coefficients estimated for ROL$_j$ and its square (see Table B.4).[43] As ROL$_j$ increases in the northeast direction, both μ^k_{hij} and μ^k_{xij} rise from a near-zero initial value to span almost the full unit interval; this holds regardless of the value of the industry characteristic (northwest axis) or of source-country rule of law (illustrated for the 10th and 90th percentile ROL$_i$ values in each row). There is a mild positive correlation between HQContractibility^k and μ^k_{hij} (Panels (a)-(b)), but no clear relationship between SSContractibility^k and μ^k_{xij} (Panels (c)-(d)). The additional plots in Figure B.5 in the appendix further confirm that ROL$_i$ in the source country does not shape μ^k_{hij} or μ^k_{xij} in a distinct way. If contracting parties tend to seek enforcement first through institutions in the country where the contract originated (i.e., the firm's home country), this could explain why ROL$_j$ exerts such influence on μ^k_{hij} and μ^k_{xij}. This role of destination-country rule of law has received less attention, largely because prior empirical work has focused on US intrafirm imports, whereas we have pooled the import and export data in our analysis.

While the estimation shows that both μ^k_{hij} and μ^k_{xij} rise with ROL$_j$, it is worth stressing that as model parameters, μ^k_{hij} and μ^k_{xij} move the propensity to integrate in opposite directions. (As we saw in Lemma 2, a higher μ^k_{hij} reduces the intrafirm trade share, since it is more important to induce

[43]This is also reflected in the positive effects estimated when we directly regress the intrafirm trade share on ROL$_j$; see the reduced-form regressions in Table B.5.

supplier effort when headquarter inputs are more contractible; the converse holds with a higher μ^k_{xij}.) We are able nevertheless to estimate the effect of ROL_j on both contractibility parameters because the polynomial specifications, $\mathbf{h}(\cdot)$ and $\mathbf{x}(\cdot)$, incorporate different industry characteristics, and can therefore tap into distinct sources of variation in the intrafirm trade data.[44] The distinct variation picked up by HQContractibility^k and SSContractibility^k is in fact evident in the reduced-form regressions where we project the intrafirm trade share directly on covariates (Table B.5, Appendix B.6): We find that the intrafirm trade share is mildly decreasing in HQContractibility^k (see the average marginal effect or inter-quartile effect, which account for the nonlinear terms in this industry variable), while increasing in SSContractibility^k. These patterns line up with the aforementioned comparative static predictions from Lemma 2.

The weighted-average values of μ^k_{hij} and μ^k_{xij} are 0.47 and 0.63 respectively (across all data points, with $\tilde{t}^k_{ijV} + \tilde{t}^k_{ijO}$ as weights). Partial contractibility is thus relevant for both headquarter and supplier tasks, with supplier tasks being slightly more contractible on average; the latter is consistent with the relatively high intrafirm trade shares seen in the data among observations with large trade volumes (see Figure 1). We discuss in detail in Appendix B.6 (in particular, with Figure B.6) how the average intrafirm trade shares in the data help to pin down these average levels of μ^k_{hij} and μ^k_{xij}; these determine in turn how much scope there is for improving welfare from removing these contracting wedges.

For the bargaining parameters, we find that the firm's share under outsourcing, β^k_{ijO}, rises over most of the range of the market power variable, Markup^k (see Figure 2(b)). That said, the implied β^k_{ijO}'s exhibit minimal variation (ranging between 0.37-0.40), suggesting that a uniform β^k_{ijO} – with a rule of thumb value of around two-fifths – would provide a reasonable approximation.[45] Indeed, when we estimate an alternative specification in which this primitive bargaining share is constant across all observations, we obtain results that are similar to our baseline (see Appendix B.7).

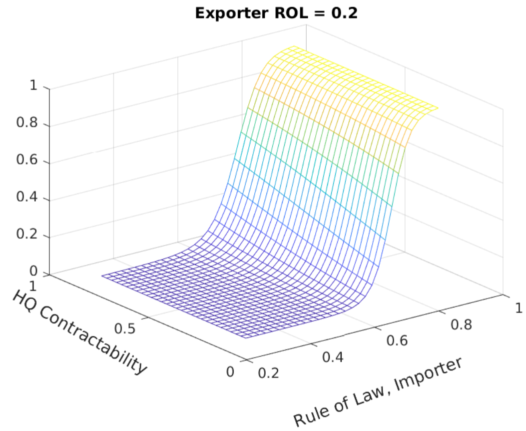
As for δ^k_{ij}, Panels (e)-(f) of Figure 3 show that this control rights parameter moves in intuitive ways with the country and industry variables it is mapped to: The share of incremental revenue the firm is assured under integration improves with the rule of law in the destination country (northeast axis), while falling in the specificity of the goods traded (northwest axis). These two variables interact meaningfully, with the lowest δ^k_{ij} values seen when the goods have high Specificity^k (making them harder to resell on open markets) and the rule of law ROL_j is weak (making it more difficult to protect against willful destruction of value).[46] The (trade-weighted) average value of δ^k_{ij} is 0.40, which implies an average firm bargaining share under integration of: $\beta^k_{ijV} = \delta^k_{ij} + (1 - \delta^k_{ij})\beta^k_{ijO} \approx 0.40 + (1 - 0.40)0.38 = 0.63$, or around two-thirds. In Appendix B.7, we also experiment with replacing the rule of law index in $\mathbf{d}(\cdot)$ with a measure of (source- and destination-country) "Recovery Rates" from the Doing Business dataset, which captures the cents on the dollar that secured creditors can typically recover in insolvency proceedings. This yields a comparable fit to the data, though our baseline specification using country of rule of law implies more conservative welfare gains from removing contracting frictions.

Sensitivity: Given the particular mappings to country and industry observables that we have used

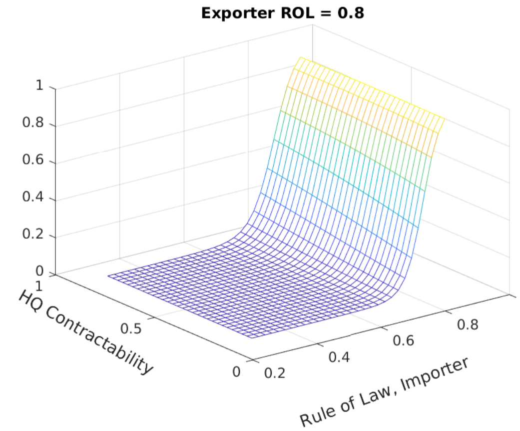
[44] More specifically, consider the partial derivatives of the intrafirm trade share in (35). A necessary condition for econometric identification is that the rows in the gradient matrix associated with the coefficient of ROL_j in $\mathbf{h}(\cdot)$ and $\mathbf{x}(\cdot)$ are linearly independent around the baseline estimate, a condition which we numerically verified.

[45] In line with this, only the constant term in the $\mathbf{b}(\cdot)$ function for β^k_{ijO} is estimated to be statistically significant at the 10% level (Table B.4, Appendix B.6); both the coefficient of Markup^k and its squared term are not precisely pinned down.

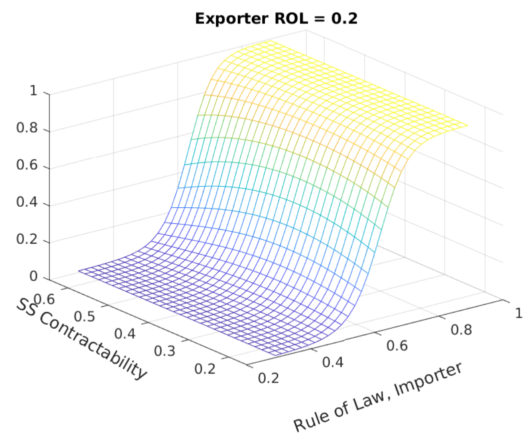
[46] While we obtain a positive point estimate for the coefficient of the BIT_{ij} dummy, this effect is not statistically significant (see Table B.4, Appendix B.6).



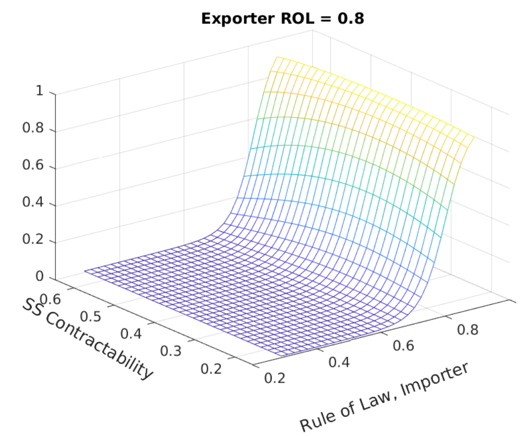
(a) μ^k_{hij}, Low Exporter ROL



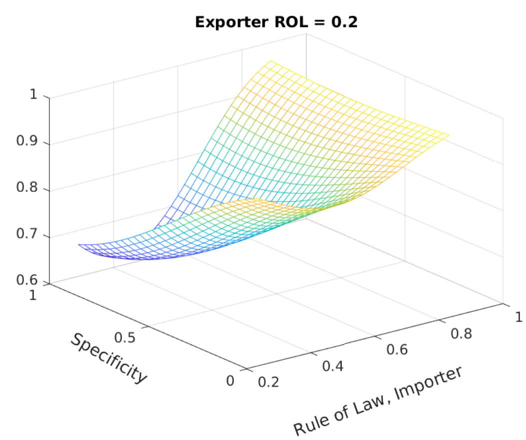
(b) μ^k_{hij}, High Exporter ROL



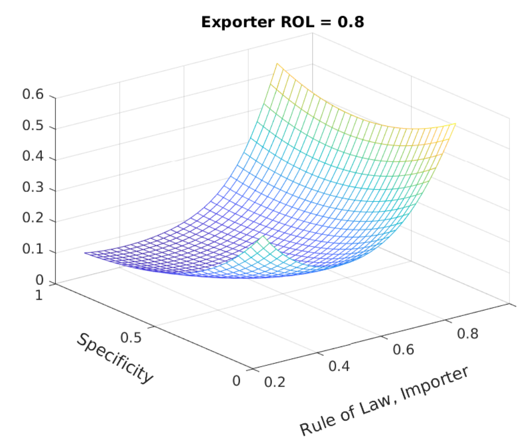
(c) μ^k_{xij}, Low Exporter ROL



(d) μ^k_{xij}, High Exporter ROL



(e) δ^k_{ij}, Low Exporter ROL



(f) δ^k_{ij}, High Exporter ROL

Figure 3: Estimated Surfaces: μ^k_{hij}, μ^k_{xij}, and δ^k_{ij}

Notes: The figures present the surface plots for μ^k_{hij}, μ^k_{xij}, and δ^k_{ij}. The plots are based on the estimated polynomial coefficients of the $\mathbf{h}(\cdot)$, $\mathbf{x}(\cdot)$, and $\mathbf{d}(\cdot)$ functions. The northeast axes plot ROL_j, while the northwest axes plot the associated industry characteristic; the left column sets ROL_i at its 10th percentile value, while the right column sets ROL_i at its 90th percentile value. The associated industry characteristics are $\text{HQContractibility}^k$ for the μ^k_{hij} surface, $\text{SSContractibility}^k$ for μ^k_{xij}, and Specificity^k for δ^k_{ij}.

to quantify our model, a natural concern is whether there might be additional relevant regressors that ought to have been considered. The concern here is with omitted variables that could be correlated with both the intrafirm trade share and an included observable, say rule of law in the importing country, as this would then bias the conclusions that we are drawing on how ROL_j is shaping key model parameters (in this case, μ^k_{hij}, μ^k_{xij}, and δ^k_{ij}). To address this, we adopt the methods advanced in Andrews et al. (2017) to perform a sensitivity analysis of our baseline parameter estimates to such omitted variables. In Appendix B.8, we describe in detail how we simulate potential omitted variables with a correlation of up to ± 0.9 with each included covariate in turn, and compute the bias in the parameter estimates that would arise from the resulting perturbation in the weighted-NLLS moment conditions. Reassuringly, the sensitivity of our baseline estimates to this concern is small. The largest potential bias we obtain stands only at 2.7% of the point estimate, this being for the coefficient on the importing country's rule of law in the $\mathbf{x}(\cdot)$ function for μ^k_{xij}. The inferred bias for the majority of the other parameters is even more muted (as summarized in Figure B.8). This reflects the good fit of the model to the data, which leaves little residual variation that could be associated with omitted variables.

5 Quantitative Implications

With these structural estimates, we can evaluate the quantitative implications of contracting frictions in global sourcing. We adopt the "hat algebra" approach (Dekle et al., 2008), which allows us to compute the proportional change, $\widehat{X} \equiv X'/X$, in outcomes X of interest in a counterfactual scenario without solving for the new equilibrium in levels. The equilibrium system of equations for our multi-country, multi-industry model (described in Section 3.3 and Appendix A.3) can be readily rewritten in changes; this hat algebra system is presented in full in equations (C.1)-(C.12) in Appendix C.1. With this, we can solve for the "hat" changes in response to an exogenous shift in trade costs (d^k_{ij}), contracting frictions (μ^k_{hij}, μ^k_{xij}), and/or bargaining shares ($\beta^k_{ij\chi}$).

We require several additional items of information to operationalize the hat algebra equations. For the π^k_{ij}'s in the initial equilibrium, we draw on the 2005 OECD Inter-Country Input-Output (ICIO) Tables for trade flow data for the full matrix of countries and industries. We work here with a set of 41 countries plus a rest of the world (ROW) composite, this being the largest overlap between the samples in the ICIO Tables and that from our estimation (see Table B.3, Appendix B.4, for the country list). The ISIC industry codes in the ICIO Tables also map naturally to NAICS 3-digit industries (as detailed in Table B.2). Recall, however, that the π^k_{ij}'s in the model are sourcing shares based on counts of products, whereas the ICIO Tables report trade flows by value; this discrepancy arises because the prices of traded inputs in our model are not determined in competitive markets. We therefore derive a model-consistent correction term in Appendix C.2 that enables us to map observed trade flow values to the π^k_{ij}'s in the model. This correction term depends only on parameters that have already been estimated, and so this does not present a major complication to the hat algebra procedure.[47]

For country-level expenditure, E_j, we read this off directly from aggregate final demand in the ICIO

[47]Specifically, we show that: $(t^k_{ij}/\varsigma^k_{ij})/(\sum_{i'=1}^{J}(t^k_{i'j}/\varsigma^k_{i'j})) = \pi^k_{ij}$, where the t^k_{ij}'s are trade flows (by value) from country i to j in industry k, and the associated correction term is:

$$\varsigma^k_{ij} = \sum_{\chi \in \{V,O\}} \left(\mu^k_{xij} + (1-\mu^k_{xij})(1-\beta^k_{ij\chi})(\zeta^k_{ij}/\zeta^k_{ij\chi})\right)\left(B^k_{ij\chi}/B^k_{ij}\right)^{\frac{\theta^k}{1-\lambda_i}}.$$

Tables; we normalize this expenditure in the US to unity and use it as the numeraire. To solve for "hat" changes, we further need information on the total payments to factors, $w_j\bar{L}_j$ and $s_j\bar{K}_j$, as well as on the trade deficit, D_j, in the baseline equilibrium. In Appendix C.3, we show that one can invert the model to infer the initial values of $\{w_j\bar{L}_j, s_j\bar{K}_j, D_j\}$ from the equilibrium conditions in equations (32)-(34). We hold D_j as fixed and exogenous throughout, following Dekle et al. (2008).

Lastly, we calibrate several upper-tier parameters related to final-good demand and production. These do not enter the expression for the intrafirm trade share in (35) and were therefore not involved in the estimation in Section 4. The demand parameter, ρ, is set to 0.80; this implies an elasticity of substitution of 5 between final-good varieties, within the plausible range in Anderson and van Wincoop (2004) and Head and Mayer (2014). The share of labor in final assembly, α, is set to 0.22; together with the labor used in supplier tasks, this yields a labor share in total output in the US of 55%, in line with Karabarbounis and Neiman (2014).[48] The η^k exponents in the final-good production function in (2) are computed as: $\eta^k = \text{INT}^k / (\sum_{k'=1}^{K} \text{INT}^{k'})$, where INT^k is the value of industry k's use as an input summed across all using manufacturing industries (drawn from the ICIO Tables).[49]

We proceed to apply the parameters estimated from US data to the global system of countries assembled from the ICIO Tables. This will allow us to gauge the quantitative impact of contracting frictions, under the premise that the US-based parameters adequately depict too the sourcing decision problem of firms in other countries. This is a data-driven limitation, as our quantification strategy could certainly incorporate intrafirm trade shares from other countries, should such data become available, in the estimation. With this caveat, we turn to the series of counterfactual exercises.

5.1 Gains from Removing Contracting Frictions

Full contractibility: How much do contracting frictions in global sourcing matter for aggregate outcomes? To address this, we simulate the counterfactual world with full contractibility by raising μ^k_{hij} and μ^k_{xij} to 1 for all i, j, and k in our model. While an environment with complete contracts would be challenging to attain in practice, this exercise is nevertheless useful for providing an upper bound on the welfare costs imposed by prevailing contracting conditions in global sourcing.

Welfare rises in all countries with this move to full contractibility (Row 1, Table 3(a)). The average welfare increase across countries is 9.2% (geometric mean), while the median is 10.1%.[50] These gains from removing contracting frictions in global sourcing are substantial; to put things in perspective, the 9.2% average welfare gain is equivalent to that from a 19.3% uniform improvement in supplier technologies (i.e., from raising the T^k_j nested-Fréchet scale parameters) across all industries and countries.[51] Not surprisingly, countries with lower initial average levels of contractibility in the baseline equilibrium, for either headquarter or supplier tasks (μ^k_{hij} or μ^k_{xij}), reap larger gains in welfare terms when contracting

[48] The labor share in our model is computed as $w_j\bar{L}_j/(E_j - D_j)$.

[49] Our analysis focuses on contracting frictions in the global sourcing of manufacturing inputs, as much of the literature has done, due to the lack of information to discipline contracting frictions in the sourcing of non-manufacturing inputs, particularly services. We *de facto* treat non-manufacturing industries as non-tradables, but account for final demand for output from these industries (as recorded in the ICIO Tables) in country expenditures, E_j.

[50] We report the geometric mean as this will allow for a straightforward exact decomposition of the sources of this change in Table 3. The arithmetic mean across countries is similar at 9.3%.

[51] Following Levchenko and Zhang (2016), the state of technology accessible to suppliers in a country-industry cell is $(T^k_j)^{1/\theta^k}$, which can be aggregated to the country level via the weighted geometric mean: $\Pi^K_{k=1}(T^k_j)^{\eta^k/\theta^k}$. We then simulate $\Pi^K_{k=1}(\widehat{T})^{\eta^k/\theta^k}$ for progressively larger values of the uniform improvement $\widehat{T}$, until we match the average country welfare gain of 9.2% in our full contractibility counterfactual.

frictions are eliminated (see Figure C.1, Appendix C.4).

	$\widehat{U_j}$	Component terms involving: $\widehat{\pi^k_{jj}}$	$\widehat{B^k_{jj}}$	$\widehat{\Upsilon^k_j}$	$\widehat{\varpi_j}$	$\widehat{w_j}$, $\widehat{s_j}$, $\widehat{E_j}$
(1): $\mu^k_{hij} = \mu^k_{xij} = 1$	0.0922	-0.0005	0.2260	0.1002	-0.0616	-0.1367
	(0.0471)	(0.0094)	(0.1537)	(0.0737)	(0.0458)	(0.1082)
(2): $\mu^k_{hij} = 1$	0.0593	-0.0009	0.1636	0.0674	-0.0468	-0.1044
	(0.0428)	(0.0056)	(0.1134)	(0.0510)	(0.0367)	(0.0871)
(3): $\mu^k_{xij} = 1$	0.0882	-0.0045	0.1979	0.0860	-0.0556	-0.1102
	(0.0438)	(0.0069)	(0.1474)	(0.0678)	(0.0439)	(0.0949)
(4): $\mu^k_{hij} = \mu^k_{xij} = 1, \lambda_i = 0$	0.1109	-0.0017	0.2435	0.1010	-0.0621	-0.1334
	(0.0540)	(0.0097)	(0.1721)	(0.0743)	(0.0462)	(0.1064)
(5): $\mu^k_{hij} = \mu^k_{xij} = 1$, free entry	0.1284	-0.0005	0.2260	0.1002	0.0366	-0.1367
	(0.0634)	(0.0094)	(0.1537)	(0.0737)	(0.0350)	(0.1082)
(6): $\mu^k_{hij} = \mu^k_{xij} = 1$, $\rho^k = 0.81$	0.0839	-0.0010	0.2118	0.1180	-0.0650	-0.1435
	(0.0419)	(0.0086)	(0.1401)	(0.0861)	(0.0464)	(0.1102)
(7): $\delta^k_{ij} = 1$ separately for each j	-0.0200	-0.0005	-0.0243	0.0016	-0.0012	0.0046
	(0.0112)	(0.0034)	(0.0130)	(0.0014)	(0.0012)	(0.0043)

(a) Decomposition of the Total Effect

	$\widehat{w_j}$	$\widehat{s_j}$	$\widehat{\varpi_j}$	$\widehat{E_j}$
(1): $\mu^k_{hij} = \mu^k_{xij} = 1$	-0.0876	0.5103	-0.2509	-0.0903
	(0.0661)	(0.4278)	(0.1691)	(0.0961)
(2): $\mu^k_{hij} = 1$	-0.1260	1.0536	-0.1957	-0.0009
	(0.0392)	(1.0976)	(0.1419)	(0.0675)
(3): $\mu^k_{xij} = 1$	0.0095	0.2252	-0.2291	-0.0507
	(0.0380)	(0.2709)	(0.1645)	(0.0725)

(b) Factor Price, Profit Share, and Aggregate Expenditure Effects

Table 3: Decomposition of Cross-Country Average Welfare Change

Notes: Panel (a) presents an exact decomposition of the average welfare change across countries based on equation (28), for the counterfactual exercises listed in each row. The first column reports the average welfare change (under the $\widehat{U_j}$ heading). The subsequent columns report averages of: $\prod_{k=1}^{K}(\widehat{\pi^k_{jj}})^{-((1-\alpha^k)/\theta^k)\eta^k(1-\alpha)}$, $\prod_{k=1}^{K}(\widehat{B^k_{jj}})^{(1-\alpha^k)\eta^k(1-\alpha)}$, $\prod_{k=1}^{K}(\widehat{\Upsilon^k_j})^{((1-\rho^k)/\rho^k)\eta^k(1-\alpha)}$, $(\widehat{\varpi_j})^{\alpha}$, and $\widehat{E_j}(\widehat{w_j})^{-\alpha}\prod_{k=1}^{K}[(\widehat{w_j})^{-(1-\alpha^k)}(\widehat{s_j})^{-\alpha^k}]^{\eta^k(1-\alpha)}$. Panel (b) presents the average "hat" changes in factor prices, the profit share, and aggregate expenditures. Each reported average is a geometric mean across countries, with the standard deviation in parentheses.

We examine the drivers behind this rise in welfare in the remaining columns of Table 3(a). Based on equation (28), $\widehat{U_j}$ can be broken down into terms that capture the role of shifts in: domestic-sourcing shares ($\widehat{\pi^k_{jj}}$), contracting capacities ($\widehat{B^k_{jj}}$), under-investment frictions ($\widehat{\Upsilon^k_j}$), the profit share ($\widehat{\varpi_j}$), as well as factor prices and aggregate expenditure ($\widehat{w_j}$, $\widehat{s_j}$, $\widehat{E_j}$); as an example, the third column in Table 3(a) reports the geometric mean of $\prod_{k=1}^{K}(\widehat{B^k_{jj}})^{(1-\alpha^k)\eta^k(1-\alpha)}$ across countries. We thus have an exact geometric decomposition, with the product of the proportional changes captured across the second through sixth columns yielding the overall welfare change in the first column.[52]

As Row 1 shows, the welfare gains in the full contractibility world are driven by the expansion in contracting capacities ($\widehat{B^k_{jj}}$) and the alleviation of under-investment frictions ($\widehat{\Upsilon^k_j}$) when contracting wedges are removed; the role of the direct-sourcing share is muted in comparison. The largest quantitative impact comes by far from the improvement in contracting capacities, which contributes a 22.6% average increase in welfare. This 22.6% number can be further decomposed log additively into: (i) the

[52]For example, in the first row, $(1 - 0.0005) \times 1.2260 \times 1.1002 \times (1 - 0.0616) \times (1 - 0.1367) = 1.0922$.

product of the average "hat" change in B^k_{jj} across industries and the average $(1-\alpha^k)\eta^k$ exponent; and (ii) the covariance between these two former terms (see Appendix C.4 for details).[53] It turns out that the former "product of averages" term contributes more than 90% of the variation in this decomposition (see Figure C.2 in the appendix). Put otherwise, the impact of the contracting wedges on sourcing capabilities is not systematically loaded on industries that have a larger elasticity in driving welfare changes. The general equilibrium adjustments in the profit share, factor prices, and aggregate expenditures offset some, but not all, of these welfare gains. As Table 3(b) reports, with complete contracting, the expansion in global sourcing raises the factor returns to capital (s_j) relative to labor (w_j); in some countries, the demand for labor even softens as there is less need to use assembly labor to mitigate under-investment in relationship-specific inputs. Improved contracting also diminishes firms' ability to extract rents through bilateral bargaining, thereby reducing the profit share; with less rebated profits, this tends to weigh down on overall expenditures (E_j).

Moving to full contractibility triggers a rich set of shifts in global trade patterns, with the resulting improvements in contracting capacities correlating strongly with increases in trade flows. We illustrate this in Figure C.3 in the appendix, where we examine third-countries' trade with China and the US. Countries that see larger improvements in their contracting capacities over goods from China compared to goods from the US (as captured by the average relative change, $(1/K)\sum_k(\widehat{B^k_{\text{CHN},j}} - \widehat{B^k_{\text{USA},j}})$, across industries) subsequently import more from China relative to the US (Figure C.3(a)); an analogous correlation holds in the pattern of countries' exports to China versus the US (Figure C.3(b)). Moreover, country institutions that facilitate contracting no longer serve as a source of comparative advantage in a world with complete contracts. This leaves technological forces and factor endowments with a greater role in shaping the industry mix of country exports; in line with this, we find for example that food manufacturing and electrical products are among the US industries that see the largest export growth with this shift to full contractibility, while it is industries such as non-metal minerals, transportation equipment, and furniture in China that expand exports the most (Figure C.4).

Headquarter vs Supplier contractibility: Do contracting frictions in headquarter or supplier tasks impose larger welfare costs? Rows 2-3 of Table 3(a) explore this natural follow-up question, by respectively raising all μ^k_{hij}'s and all μ^k_{xij}'s to 1 in separate simulations. We find that eliminating contracting frictions on only supplier tasks generates an average welfare gain of 8.8% across countries, larger than the 5.9% increase when removing these frictions only for headquarter services. Recall that supplier tasks are on average more contractible in the baseline equilibrium (with an average μ^k_{xij} of 0.63 versus 0.47 for μ^k_{hij}). This means that our finding that welfare is more responsive to improvements in μ^k_{xij} is ultimately driven by the relative importance of supplier tasks in the firm's production function (with the $1-\alpha^k$ exponent being around two-thirds).[54]

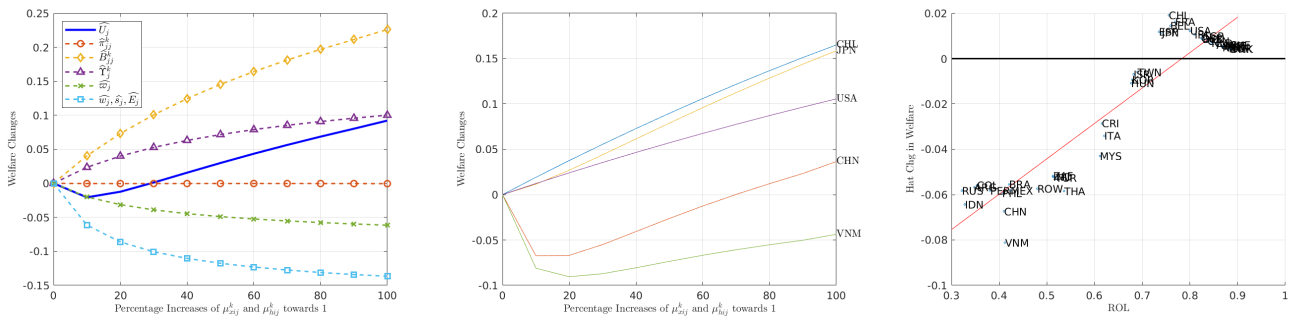
Gradual changes: We next consider more modest improvements in the contracting environment, to shed light on a trajectory of welfare gains that is more achievable in practice. We run a series of

[53]Specifically, let $\bar{\nu} \equiv \sum_{k=1}^K \nu^k/K$ be the average of the $\nu^k \equiv \eta^k(1-\alpha^k)$ exponents, and $\overline{\log \widehat{B^k_{jj}}} \equiv \sum_{k=1}^K \log\left(\widehat{B^k_{jj}}\right)/K$ denote the average change in contracting capacity across industries within a country. We have:

$$\log \prod_{k=1}^K \left(\widehat{B^k_{jj}}\right)^{\eta^k(1-\alpha^k)(1-\alpha)} = (1-\alpha)K\bar{\nu} \times \overline{\log \widehat{B^k_{jj}}} + (1-\alpha)\sum_{k=1}^K \left(\nu^k - \bar{\nu}\right)\left(\log \widehat{B^k_{jj}} - \overline{\log \widehat{B^k_{jj}}}\right).$$

[54]Not surprisingly, when contracting frictions on only supplier tasks are removed, this tends to be more beneficial for wages w_j, compared to the opposite scenario with complete contracting for only headquarter tasks (see Table 3(b)).

counterfactuals in which we close the gap between each μ_{hij}^k and its maximum value of 1 (and likewise, between each μ_{xij}^k and 1) progressively by 10%, 20%, and so on; the final simulation where 100% of the gap is closed is precisely the full contractibility world (with $\mu_{xij}^k = \mu_{hij}^k = 1$ for all i, j, and k).



(a) Decomposition (b) All Countries (c) Welfare change against initial ROL

Figure 4: Gradual Changes in Contracting Frictions

Notes: The figure simulates a series of improvements in all μ_{hij}^k's and μ_{xij}^k's that closes the gap to the full contractibility world by 10%, 20%,..., 100%. Panel (a) plots the geometric mean across countries of the "hat" change in welfare, as well as the terms following the decomposition in Table 3(a). Panel (b) plots the "hat" change in welfare for a subset of countries, to illustrate the range of outcomes spanned by CHL (largest welfare gain) and VNM (lowest). Panel (c) plots the "hat" change in welfare against countries' initial rule of law for the case of the 10% improvement.

With this stepwise reduction in contracting frictions, the response in average country welfare is, rather interestingly, not monotonic but U-shaped. Average welfare falls with the initial improvements in contractibility, and this switches to a welfare gain only after a 30% movement toward the frictionless world. This non-monotonicity is a quantitative finding, that stems from a horse race between the gains from better contracting and the offsetting effects from factor price and expenditure movements. As Figure 4(a) shows, the changes in contracting capacities $(\widehat{B_{jj}^k})$ and input investments $(\widehat{\Upsilon_j^k})$ are positive for each stepwise increase in μ_{hij}^k and μ_{xij}^k. However, similar to what we saw in Table 3(b), adjustments in factor prices, the profit share, and aggregate expenditure generate a countervailing effect, and these tend to dominate when the improvement in μ_{xij}^k and μ_{hij}^k is small. This is especially true for countries with low initial levels in the rule of law index: Figure 4(c) plots the welfare change for a 10% global reduction in contracting frictions, and shows that it is the low rule of law countries, such as Vietnam and China, that undergo a welfare loss (see also Figure 4(b)). Countries with high rule of law, such as Chile, Japan, and the US, instead benefit since they secure a greater share of global sourcing activity with these initial reductions in contracting frictions; welfare for these countries rises monotonically with each stepwise improvement toward the complete contracting world. This points to a "low institutions trap": Countries with low rule of law may not find it in their interest to coordinate small improvements in their contracting environment together with the rest of the world, even though deeper and more substantive reforms would eventually deliver welfare gains.

5.2 Further Checks and Exercises

We briefly examine the role played by several key model features in our counterfactual welfare assessments; these exercises are reported in the remaining rows of Table 3(a).

Correlated supplier draws: In Row 4, we highlight the importance of the λ_i correlation parameters in the distribution of supplier draws. We do so by conducting a full contractibility exercise while setting $\lambda_i = 0$ for all countries (and holding all other parameters at their baseline values). This delivers even larger welfare gains, averaging 11.1% across countries; absent these correlation parameters, we would be led to over-state the welfare impacts of contracting frictions in global sourcing compared to the original Row 1 counterfactual. Intuitively, imposing a zero-correlation structure raises the dispersion in each pair of supplier draws from the same source country. With the thick-tailed nature of the Fréchet distribution, this exaggerates the welfare gains from removing contracting frictions.[55]

Free entry: The welfare gains in our baseline counterfactual are also conservative compared to what we obtain when we further allow for the endogenous entry of final-good producers (Row 5). In Appendix C.5, we develop an extension in which we show how to tractably incorporate a free entry condition to pin down the mass of firms, N_j (which until now has been assumed to be exogenous and fixed). With endogenous entry, we find more sizeable welfare gains, an average increase of 12.8%, from moving to full contractibility. This is because improvements in contracting also induce more firms to enter, which boosts welfare through a familiar love-of-variety effect.

Heterogeneous elasticities: Row 6 of Table 3(a) reports on an exercise in which we neutralize the variation in ρ^k across industries, by setting this to its average value of 0.81 for all industries k. It is reassuring that heterogeneity in this key elasticity (c.f., Ossa, 2015) is not driving the welfare effects; the average country welfare gain under full contractibility is similar to that from our baseline counterfactual, albeit slightly lower at 8.4%.[56]

Gains from integration: In the last row of Table 3(a), we explore how much the organizational mode dimension – specifically, the option to vertically integrate one's suppliers – contributes in welfare terms. For this, we revert to the baseline world with contracting frictions, and instead set the control rights parameter δ_{ij}^k to its limit value of 1, so that $\beta_{ijV}^k = 1$. When a firm has full bargaining power, its integrated suppliers would be completely disincentivized from delivering inputs, and so $\chi = V$ would never be favored. Thus, this limit case de facto shuts down integration as an active mode choice. Firms would then re-optimize their sourcing decisions; for each individual variety previously procured under integration, they would select one of the J countries to source from at arm's length ($\chi = O$) instead.[57] Unlike the exercises in the prior rows, we set $\delta_{ij}^k = 1$ here country-by-country for $j = 1, 2, \cdots, 42$ (i.e., in turn for each j, we set $\delta_{ij}^k = 1$ for all i and k). This is because the welfare impact of vertical integration turns out to be fairly mild, and so general equilibrium factor price and expenditure responses can obscure interesting patterns if the organizational mode were removed globally.

When integration is shut off for a country, this decreases its contracting capacity ($\widehat{B_{jj}^k} < 0$), which weighs down on welfare. Across the 42 separate simulations, the average welfare change for the affected country is a modest loss of 0.02% (Row 7). As one might expect, the loss is larger for countries that relied more on integration: there is a negative correlation of -0.47 (-0.65) between the intrafirm import (export) share exhibited by a country in the initial equilibrium and the subsequent welfare change (see Figure C.6, Appendix C.6). For the specific case of the US, the gains from integration we obtain

[55] Figure C.5 in the appendix shows that, comparing the $\lambda_i = 0$ scenario (Row 4) with the baseline counterfactual (Row 1), the welfare gains from full contractibility are over-stated particularly for countries with low initial rule of law.

[56] Raising the value of this uniform ρ^k above 0.81 magnifies the welfare gains from removing contracting frictions. As input varieties become more substitutable, this exacerbates under-investment in both headquarter and supplier tasks, which implies larger gains from eliminating these contracting frictions.

[57] To see this, note that the probability of sourcing under integration, $\pi_{V|ij}^k$, in equation (20) is equal to 0 when $\delta_{ij}^k = 1$; this is because $\beta_{ijV}^k = 1$ and $B_{ijV}^k = 0$ in this special case.

amount to 0.3%. Interestingly, this is similar to the gains from vertical integration of 0.23% in Garetto (2013), although the underlying model mechanisms are different; we focus on how integration provides better incentives for the firm to invest in relationship-specific inputs, while integration in Garetto (2013) allows the firm to lower the markups paid for inputs and to partially transfer technology. On the other hand, the welfare impacts we find are smaller than the "gains from multinational production" reported elsewhere in the quantitative trade literature (Ramondo and Rodríguez-Clare, 2013; Ramondo, 2014; Arkolakis et al., 2018; Alviarez, 2019); note though that these papers incorporate both vertical and horizontal FDI, whereas our model features only vertical integration.[58]

5.3 Implications for the Gains from Trade

We next use our model to study the interplay between contracting frictions and trade costs: Does the severity of contracting frictions have any bearing on the size of the gains from trade?

To fix ideas, we calculate the gains from trade as the "hat" change in welfare in response to a one percent reduction to all cross-border iceberg trade costs (i.e., all d_{ij}^k's with $i \neq j$).[59] We compare this "welfare elasticity" under two settings: (i) the baseline world with contracting frictions (i.e., with the μ_{hij}^k's and μ_{xij}^k's pinned down from the structural estimation); and (ii) the counterfactual world with full contractibility (i.e., $\mu_{hij}^k = \mu_{xij}^k = 1$ for all i, j, and k). Figure 5(a) plots the difference (in percentage terms) between these gains from trade in the latter relative to the former scenario; as an example, the 1.18 value for "JPN" means that the welfare response to this change in trade costs is 118% higher for Japan under full contractibility versus the baseline equilibrium.[60] Put slightly differently, this captures the extent to which the conventional Arkolakis et al. (2012) formula in equation (29) would over-state the gains from trade compared to the more general welfare change expression we derived in (28) that incorporates contracting frictions. To help with intuition, we also consider how the average absorption ratio (the average $\widehat{\pi_{jj}^k}$ across industries) responds to this one percent change in trade costs; specifically, we illustrate in Figure 5(b) the analogous differential response (in percentage terms) in this average absorption ratio across the two contracting scenarios.[61]

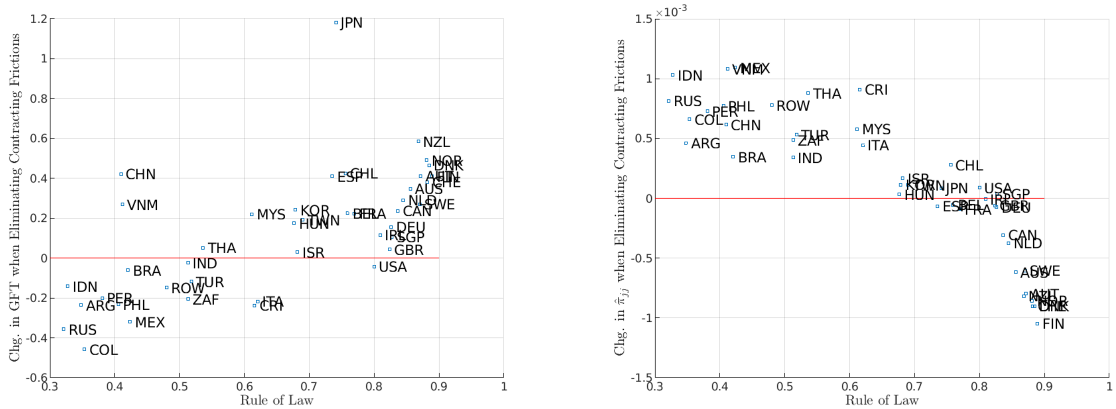
Importantly, we find that the gains from trade are misstated if one were to (incorrectly) presume a world with full contractibility. The extent of this bias is nontrivial and its direction is moreover not uniform across countries, which underscores the need for a careful quantitative assessment. For countries above the zero-change line in Figure 5(a), the gains from trade are over-stated relative to the factual world with contracting frictions, by an average of around 31%. In words, these are countries that especially benefit from reductions in trade costs if contracting were frictionless. This group includes high rule of law developed countries (e.g., Japan, Norway) that appear toward the right of Figure 5(a) and 5(b). A reduction in trade costs induces these countries to expand their foreign sourcing, and if the world featured full contractibility, this would include an increase in sourcing from locations with

[58]Atalay et al. (2019) and Eppinger and Ma (2024) find that vertical integration is associated with large increases in productivity and shipments, although these documented gains at the firm level are not straightforward to translate to aggregate implications.

[59]We obtain qualitatively similar results when evaluating the gains relative to autarky (i.e., $d_{ij}^k \to \infty$ for all $i \neq j$).

[60]More specifically, a 1% reduction in cross-border trade costs raises welfare by 0.0412% for Japan in the full contractibility world, and by just 0.0189% in the factual world with contracting frictions.

[61]We calculate the average of $\widehat{\pi_{ii}^k}$ across all industries k, using the initial domestic sourcing value of each industry as weights. The vertical axis in Figure 5(b) then plots the change in percentage terms in this average absorption ratio in the full contractibility world relative to the baseline world with contracting frictions.



(a) Gains from Trade

(b) Domestic Absorption Ratio

Figure 5: Gains from Trade under Full Contractibility versus the Baseline with Contracting Frictions

Notes: Panel (a) plots the difference, expressed in percentage terms, in the gains from trade (from a 1% reduction in cross-border trade costs) under full contractibility relative to the baseline with contracting frictions. Panel (b) plots the corresponding differential response across these scenarios in the domestic absorption ratio; the latter is calculated as a weighted average across industries, using industries' domestic sourcing value as weights. Both variables are arrayed against country rule of law on the horizontal axis.

low initial levels of the rule of law. For these developed countries, this generates a larger decrease in domestic sourcing shares (as illustrated in Figure 5(b) by the average absorption ratio), and thus an over-stating of the welfare gains from trade relative to the factual world with contracting wedges. At the same time, there are several notable developing countries (e.g., China, Malaysia, Vietnam) for whom a more secure contracting environment coupled with a trade cost reduction would significantly expand their role as global suppliers, due in particular to their large labor endowments. In a world that is presumed to have frictionless contracting, these countries' gains from trade are amplified too, due to the rise in demand for their labor and the resulting positive income shock.

Conversely, for countries that sit below the zero-change line in Figure 5(a), the gains from trade are under-stated, by an average of around 20%, when contracting frictions are ignored. This group comprises mainly countries with low rule of law (e.g., Indonesia, Russia), and these institutional conditions also impede their sourcing of inputs from domestic suppliers. In a hypothetical world with full contractibility, these countries' firms would want to engage relatively more with domestic suppliers, and hence a reduction in trade costs results in a smaller decrease in the domestic sourcing share compared to the factual world with contracting frictions (see Figure 5(b)). This leads to the more muted welfare gains from trade for these countries when full contractibility is assumed.

5.4 US-China Decoupling

As a last counterfactual, we consider the impact of a decoupling in US-China trade relations, the prospect of which has been raised by renewed geopolitical and economic tensions. This serves as a counterpart to the exercise in the previous section, to instead consider a scenario with elevated trade barriers. To simulate this decoupling, we set the iceberg trade costs between the US and China to infinity for all industries, while leaving the d_{ij}^k's involving other countries unchanged.

Table 4 reports the welfare effects from the resulting cessation in bilateral trade between the US and China. In the baseline world with contracting frictions (left column), the US and China both experience welfare losses with this trade decoupling. The impact on the US is relatively small (a dip in welfare of just 0.16%), due in large part to the US' relatively high domestic absorption share π^k_{jj} across industries; losing China as a supplier only slightly raises prices in the US, so the fall in US real income is milder.[62] By contrast, China undergoes a much larger welfare loss (2.52%), as the unwinding of access to the US market results in sharp declines in factor prices in China. As for the rest of the world, the average welfare loss across other countries is fairly negligible (0.003%). There are however notable examples of individual countries that see positive spillovers. Vietnam, in particular, benefits the most (with a welfare gain of 0.34%), as it partially replaces China's market share in the US; interestingly, this is in line with the actual shifts in global trade patterns seen since 2017 with the escalation in US-China tensions (Alfaro and Chor, 2023; Freund et al., 2024; Grossman et al., 2024).

Welfare Change (%)	Baseline Contracting Frictions	Full Contractibility
Welfare Change in USA	-0.1613	-0.2352
Welfare Change in China	-2.5240	-2.7686
(Average) Welfare Change in ROW	-0.0031	0.0484

Table 4: The Welfare Impacts of US-China Decoupling

Notes: This table reports the welfare impact of the US-China decoupling counterfactual, in which we increase the trade costs between the two countries in all industries to infinity. The "baseline contracting frictions" case uses the μ^k_{hij}'s and μ^k_{xij}'s implied by our structural estimation, while the "full contractibility" case sets $\mu^k_{hij} = \mu^k_{xij} = 1$ for all i, j, and k.

Our quantitative model allows us to further assess whether contracting frictions affect these welfare losses in a meaningful way. Indeed, we find that the welfare losses from US-China decoupling would be more costly for both countries in a world with full contractibility (Table 4, right column): Welfare in the US and China falls by 0.24% and 2.77% respectively, due to the more extensive sourcing links between the two countries in a world devoid of contracting frictions. This is much in line with the findings from Section 5.3, where we uncovered an amplification in the welfare gains from trade under full contractibility among countries that were initially well-engaged in cross-border sourcing already. An upshot here is that a strong contracting environment could provide some bulwark to deter countries from the threat of decoupling, by raising the welfare stakes for both the US and China from severing their trade links.

6 Conclusion

In this paper, we have developed a model of global sourcing in the shadow of incomplete contracts, that enables us to address the aggregate implications of these contracting frictions. The work here builds a bridge between firm-level models of ownership boundaries grounded in the property-rights approach of Grossman-Hart-Moore, and general equilibrium models of production and trade; this has been achieved by applying and innovating on tools from the quantitative trade literature. The approach we implement has a number of appealing methodological features: analytical expressions in which contracting frictions enter as iceberg-like wedges; the relative ease of simulating the full contractibility world as the limit

[62] This is comparable to the 0.12% welfare loss from a unilateral US decoupling from China in Eppinger et al. (2021).

case of a parsimonious set of model parameters; the derivation of a structural estimating equation for the intrafirm trade share; and relatively low data requirements for quantification.

While this has entailed investing in modeling structure, what we gain in return is the ability to provide model-based insights on the aggregate influence of contracting frictions in global sourcing. We find an average improvement across countries of 9.2% in welfare terms if one could eliminate these frictions, which is equivalent to the welfare gain from a 19.3% uniform improvement in supplier technologies. Contracting frictions are thus not only conceptually relevant, but also quantitatively sizeable. Moreover, failing to account for these contracting frictions can lead to inaccurate assessments of the gains from trade, with the sign and size of this bias varying widely across countries.

There clearly remain avenues for future work, of which we highlight two below. As it stands, our model features a short production chain; future extensions could explore the implications of contracting frictions in global value chains with multiple stages (i.e., a longer "snake"), in which sourcing exhibits a more meaningful sequentiality. On a separate note, while we have pursued a quantification strategy that relies on aggregated, publicly available data on intrafirm trade shares, a natural extension (already alluded to at the end of Section 2) would be to bring this framework to firm-level data, to shed light on potential heterogeneity in firms' responses to contracting frictions in global sourcing.

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A Model: Derivations and Proofs

A.1 Proofs from Section 2

Incremental revenue: Suppose that the number of input varieties in industry k is finite and equal to $\mathcal{L}$, with each input variety being of measure $\epsilon = 1/\mathcal{L}$, so that the input varieties are collectively of measure 1. With a slight abuse of notation, we use l to refer to these input varieties where $1 \leq l \leq \mathcal{L}$. (We omit the industry superscript on l to reduce notation in the proof below.) We seek to compute the incremental contribution to firm revenue arising from a particular l. Consistent with the "Nash-in-Nash" approach, the assembly labor used by the firm, $L_j(\phi)$, and the headquarter and supplier services embodied in all other input varieties – those in industry k other than l, as well as all input varieties in all other industries outside of k – are taken as given.

When the input variety l is included in the production process, the value of firm revenue is equal to:

$$\widetilde{r}^k_{\mathrm{IN},j}(l;\phi) = A_j^{1-\rho}\phi^{\rho}L_j(\phi)^{\alpha\rho}\left[\prod_{k'\neq k}\left(X_j^{k'}(\phi)\right)^{\eta^{k'}(1-\alpha)\rho}\right]\left[\tilde{x}^k_j(l;\phi)^{\rho^k}\epsilon + \sum_{l'\neq l}\tilde{x}^k_j(l';\phi)^{\rho^k}\epsilon\right]^{\frac{\eta^k(1-\alpha)\rho}{\rho^k}}, \tag{A.1}$$

bearing in mind that each of the $\mathcal{L}$ inputs is of measure ϵ. Without input variety l, we instead have:

$$\widetilde{r}^k_{\mathrm{OUT},j}(l;\phi) = A_j^{1-\rho}\phi^{\rho}L_j(\phi)^{\alpha\rho}\left[\prod_{k'\neq k}\left(X_j^{k'}(\phi)\right)^{\eta^{k'}(1-\alpha)\rho}\right]\left[\sum_{l'\neq l}\tilde{x}^k_j(l';\phi)^{\rho^k}\epsilon\right]^{\frac{\eta^k(1-\alpha)\rho}{\rho^k}}. \tag{A.2}$$

Combining (A.1) and (A.2), the incremental revenue generated by input variety l, $\widetilde{r}(l,\epsilon) = \widetilde{r}_{\mathrm{IN}} - \widetilde{r}_{\mathrm{OUT}}$, is:

$$\widetilde{r}(l,\epsilon) = A_j^{1-\rho}\phi^{\rho}L_j(\phi)^{\alpha\rho}\left[\prod_{k'\neq k}\left(X_j^{k'}(\phi)\right)^{\eta^{k'}(1-\alpha)\rho}\right]\left\{\left[\tilde{x}^k_j(l;\phi)^{\rho^k}\epsilon + \sum_{l'\neq l}\tilde{x}^k_j(l';\phi)^{\rho^k}\epsilon'\right]^{\frac{\eta^k(1-\alpha)\rho}{\rho^k}} - \left[\sum_{l'\neq l}\tilde{x}^k_j(l';\phi)^{\rho^k}\epsilon'\right]^{\frac{\eta^k(1-\alpha)\rho}{\rho^k}}\right\},$$

where we denote the measure of each input variety $l' \neq l$ by ϵ' to distinguish this from ϵ (which denotes the measure of input variety l). Following Acemoglu et al. (2007), we now approximate the term in the curly braces from this previous equation via a first-order Taylor expansion about $\epsilon = 0$. Since $\widetilde{r}(l,0) = 0$, we have:

$$\begin{aligned}\frac{\widetilde{r}(l,\epsilon)}{\epsilon} &\approx \frac{\widetilde{r}(l,0)}{\epsilon} + \left.\frac{\partial\widetilde{r}(l,\epsilon)}{\partial\epsilon}\right|_{\epsilon=0} \\ &= A_j^{1-\rho}\phi^{\rho}L_j(\phi)^{\alpha\rho}\left[\prod_{k'\neq k}\left(X_j^{k'}(\phi)\right)^{\eta^{k'}(1-\alpha)\rho}\right]\left[\sum_{l'\neq l}\tilde{x}^k_j(l';\phi)^{\rho^k}\epsilon'\right]^{\frac{\eta^k(1-\alpha)\rho}{\rho^k}-1}\frac{\eta^k(1-\alpha)\rho}{\rho^k}\tilde{x}^k_j(l;\phi)^{\rho^k}.\end{aligned}$$

As $\mathcal{L} \longrightarrow \infty$, we have $\epsilon = \epsilon' = 1/\mathcal{L} \longrightarrow 0$, and the summation term in the above equation becomes a Riemann integral. The incremental revenue contribution from input variety $l^k \in [0,1]$ is thus:

$$\begin{aligned}r^k_j(l^k;\phi) = \lim_{\mathcal{L}\to\infty}\frac{\widetilde{r}(l^k,\epsilon)}{\epsilon} &= A_j^{1-\rho}\phi^{\rho}L_j(\phi)^{\alpha\rho}\left[\prod_{k'\neq k}\left(X_j^{k'}(\phi)\right)^{\eta^{k'}(1-\alpha)\rho}\right]\left[\int_{l'=0}^{1}\tilde{x}^k_j(l';\phi)^{\rho^k}dl'\right]^{\frac{\eta^k(1-\alpha)\rho}{\rho^k}-1}\frac{\eta^k(1-\alpha)\rho}{\rho^k}\tilde{x}^k_j(l^k;\phi)^{\rho^k} \\ &= A_j^{1-\rho}\phi^{\rho}L_j(\phi)^{\alpha\rho}\left[\prod_{k'=1}^{K}\left(X_j^{k'}(\phi)\right)^{\eta^{k'}(1-\alpha)\rho}\right]\left(X_j^k(\phi)\right)^{-\rho^k}\frac{\eta^k(1-\alpha)\rho}{\rho^k}\tilde{x}^k_j(l^k;\phi)^{\rho^k} \\ &= (1-\alpha)\frac{\rho\eta^k}{\rho^k}R_j(\phi)\left(\frac{\tilde{x}^k_j(l^k;\phi)}{X^k_j(\phi)}\right)^{\rho^k},\end{aligned}$$

where recall that the revenue of the firm is $R_j(\phi) = A_j^{1-\rho}q_j(\phi)^{\rho} = A_j^{1-\rho}y_j(\phi)^{\rho}$. This yields the expression for the incremental revenue contribution reported in equation (7) in the main text.

Noncontractible tasks: Recall that the firm and supplier for l^k choose their respective investments in noncontractible task effort, taking as given the noncontractible investments of the other party, as well as the contractible task levels and the sourcing mode that have been written down in the initial contract. From the objective function in (8), the first-order condition for $h_j^k(\iota_h;\phi,l^k)$ for all $\iota_h \in (\mu_{hij}^k,1]$ is:

$$h_j^k(\iota_h;\phi,l^k) = \beta_{ij\chi}^k \frac{(1-\alpha)\rho\eta^k R_j(\phi)}{\left(X_j^k(\phi)\right)^{\rho^k}} \left(\exp\left[\int_0^{\mu_{hij}^k} \log h_j^k(\iota_h';\phi,l^k)d\iota_h'\right]\right)^{\rho^k\alpha^k} \left(x_j^k(l^k;\phi)\right)^{\rho^k(1-\alpha^k)}$$
$$\times\, \alpha^k \left(\exp\left[\int_{\mu_{hij}^k}^1 \log h_j^k(\iota_h';\phi,l^k)d\iota_h'\right]\right)^{\alpha^k\rho^k} \frac{1}{s_j}.$$

Inspecting this last equation, the right-hand side does not depend on the specific identity of the noncontractible task input $\iota_h \in (\mu_{hij}^k,1]$ that is being considered. The optimal $h_j^k(\iota_h;\phi,l^k)$ is thus identical for all $\iota_h \in (\mu_{hij}^k,1]$, and we denote this common level of investment in noncontractible headquarter tasks by $h_{nj}^k(l^k;\phi)$. Substituting this property into the above first-order condition, we obtain:

$$h_{nj}^k(l^k;\phi)^{1-\rho^k\alpha^k(1-\mu_{hij}^k)} = \frac{(1-\alpha)\rho\eta^k R_j(\phi)}{\left(X_j^k(\phi)\right)^{\rho^k}} \frac{\alpha^k\beta_{ij\chi}^k}{s_j} \left(\exp\left[\int_0^{\mu_{hij}^k} \log h_j^k(\iota_h';\phi,l^k)d\iota_h'\right]\right)^{\rho^k\alpha^k} x_j^k(l^k;\phi)^{\rho^k(1-\alpha^k)}. \quad \text{(A.3)}$$

This expresses the firm's investment level in each noncontractible headquarter task ($\iota_h \in (\mu_{hij}^k,1]$), conditional on the supplier task inputs and the pre-specified investments for the contractible headquarter tasks ($\iota_h \in [0,\mu_{hij}^k]$).

We solve for the supplier's choice of $x_j^k(\iota_x;\phi,l^k)$ for all its noncontractible tasks $\iota_x \in (\mu_{xij}^k,1]$ in a similar manner. The associated first-order condition based on the supplier's objective function in (9) is:

$$x_j^k(\iota_x;\phi,l^k) = \left(1-\beta_{ij\chi}^k\right) \frac{(1-\alpha)\rho\eta^k R_j(\phi)}{\left(X_j^k(\phi)\right)^{\rho^k}} \left(h_j^k(l^k;\phi)\right)^{\rho^k\alpha^k} \left(\exp\left[\int_0^{\mu_{xij}^k} \log x_j^k(\iota_x';\phi,l^k)d\iota_x'\right]\right)^{\rho^k(1-\alpha^k)}$$
$$\times \left(1-\alpha^k\right) \left(\exp\left[\int_{\mu_{xij}^k}^1 \log x_j^k(\iota_x';\phi,l^k)d\iota_x'\right]\right)^{(1-\alpha^k)\rho^k} \frac{1}{c_{ij\chi}^k(l^k;\phi)}.$$

Once again, this last equation implies that the supplier for input variety l^k will choose an identical investment level for each of the noncontractible supplier tasks, so that $x_j^k(\iota_x;\phi,l^k) = x_{nj}^k(l^k;\phi)$ for all $\iota_x \in (\mu_{xij}^k,1]$. Substituting this back into the first-order condition and simplifying, we obtain:

$$x_{nj}^k(l^k;\phi)^{1-\rho^k(1-\alpha^k)(1-\mu_{xij}^k)} = \frac{(1-\alpha)\rho\eta^k R_j(\phi)}{\left(X_j^k(\phi)\right)^{\rho^k}} \frac{(1-\alpha^k)(1-\beta_{ij\chi}^k)}{c_{ij\chi}^k(l^k;\phi)} \left(\exp\left[\int_0^{\mu_{xij}^k} \log x_j^k(\iota_x';\phi,l^k)d\iota_x'\right]\right)^{\rho^k\left(1-\alpha^k\right)} h_j^k(l^k;\phi)^{\rho^k\alpha^k}, \quad \text{(A.4)}$$

which expresses the supplier's investment level in each noncontractible task ($\iota_x \in (\mu_{xij}^k,1]$) as a function of the headquarter task inputs and the pre-specified investments for the contractible supplier tasks ($\iota_x \in [0,\mu_{xij}^k]$).

We now simplify (A.3) and (A.4), in order to express $h_{nj}^k(l^k;\phi)$ and $x_{nj}^k(l^k;\phi)$ as functions just of contractible task investment levels. Dividing (A.3) by (A.4), one obtains:

$$h_{nj}^k(l^k;\phi) = \frac{\beta_{ij\chi}^k}{1-\beta_{ij\chi}^k} \frac{\alpha^k}{1-\alpha^k} \frac{c_{ij\chi}^k(l^k;\phi)}{s_j} x_{nj}^k(l^k;\phi). \quad \text{(A.5)}$$

We substitute this expression for $h_{nj}^k(l^k;\phi)$ into (A.4), bearing in mind from equation (5) in the main paper that $h_j^k(l^k;\phi)$ is a function of $h_{nj}^k(l^k;\phi)$. After some algebraic simplification, we obtain:

$$x_{nj}^k(l^k;\phi)^{\zeta_{ij}^k} = \frac{(1-\alpha)\rho\eta^k R_j(\phi)}{\left(X_j^k(\phi)\right)^{\rho^k}} \times \left(\exp\left[\int_0^{\mu_{hij}^k} \log h_j^k(\iota_h;\phi,l^k)d\iota_h\right]\right)^{\rho^k\alpha^k} \left(\exp\left[\int_0^{\mu_{xij}^k} \log x_j^k(\iota_x;\phi,l^k)d\iota_x\right]\right)^{\rho^k\left(1-\alpha^k\right)}$$
$$\times \left(\frac{\alpha^k\beta_{ij\chi}^k}{s_j}\right)^{\rho^k\alpha^k\left(1-\mu_{hij}^k\right)} \left(\frac{\left(1-\alpha^k\right)\left(1-\beta_{ij\chi}^k\right)}{c_{ij\chi}^k(l^k;\phi)}\right)^{1-\rho^k\alpha^k\left(1-\mu_{hij}^k\right)}, \quad \text{(A.6)}$$

where recall that $\zeta_{ij}^k = 1-\rho^k\alpha^k(1-\mu_{hij}^k)-\rho^k(1-\alpha^k)(1-\mu_{xij}^k)$, from equation (14) in the main paper.

On the other hand, rearranging (A.5), $x^k_{nj}(l^k;\phi)$ can be re-written as a function in particular of $h^k_{nj}(l^k;\phi)$; we plug this expression for $x^k_{nj}(l^k;\phi)$ into equation (A.3) and simplify to obtain:

$$h^k_{nj}(l^k;\phi)^{\zeta^k_{ij}} = \frac{(1-\alpha)\rho\eta^k R_j(\phi)}{\left(X^k_j(\phi)\right)^{\rho^k}} \times \left(\exp\left[\int_0^{\mu^k_{hij}} \log h^k_j(\iota_h;\phi,l^k)d\iota_h\right]\right)^{\rho^k\alpha^k} \left(\exp\left[\int_0^{\mu^k_{xij}} \log x^k_j(\iota_x;\phi,l^k)d\iota_x\right]\right)^{\rho^k(1-\alpha^k)}$$
$$\times \left(\frac{\alpha^k\beta^k_{ij\chi}}{s_j}\right)^{1-\rho^k(1-\alpha^k)(1-\mu^k_{xij})} \left(\frac{(1-\alpha^k)\left(1-\beta^k_{ij\chi}\right)}{c^k_{ij\chi}(l^k;\phi)}\right)^{\rho^k(1-\alpha^k)(1-\mu^k_{xij})} \tag{A.7}$$

Equations (A.6) and (A.7) thus yield the investment levels for noncontractible supplier and headquarter tasks respectively, expressed as a function of the contractible task investments.

Contractible input tasks: Working backwards along the timeline of events, we now solve for the investment levels in contractible tasks that would be spelled out by the firm in the initial contract. In the presence of ex-ante transfers, these contractible task levels are chosen to maximize the joint surplus from the bilateral interaction with the supplier for l^k, given by (11) in the main text. We substitute into (11) the expressions for $h^k_{nj}(l^k;\phi)$ from (A.7) and for $x^k_{nj}(l^k;\phi)$ from (A.6); we do so wherever these noncontractible tasks appear either in the incremental revenue or the cost terms in (11). After some algebra, $F^k_{ij\chi}(l^k;\phi)$ can be re-written as:

$$\frac{1}{\rho^k}\zeta^k_{ij\chi}\left(\frac{(1-\alpha)\rho\eta^k R_j(\phi)}{(X^k_j(\phi))^{\rho^k}}\right)^{\frac{1}{\zeta^k_{ij}}} \left(\exp\left[\int_0^{\mu^k_{hij}} \log h^k_j(\iota_h;\phi,l^k)d\iota_h\right]\right)^{\frac{\rho^k\alpha^k}{\zeta^k_{ij}}} \left(\exp\left[\int_0^{\mu^k_{xij}} \log x^k_j(\iota_x;\phi,l^k)d\iota_x\right]\right)^{\frac{\rho^k(1-\alpha^k)}{\zeta^k_{ij}}}$$
$$\times \left(\frac{\alpha^k\beta^k_{ij\chi}}{s_j}\right)^{\frac{\rho^k\alpha^k(1-\mu^k_{hij})}{\zeta^k_{ij}}} \left(\frac{(1-\alpha^k)\left(1-\beta^k_{ij\chi}\right)}{c^k_{ij\chi}(l^k;\phi)}\right)^{\frac{\rho^k(1-\alpha^k)(1-\mu^k_{xij})}{\zeta^k_{ij}}} - s_j\int_0^{\mu^k_{hij}} h^k_j(\iota_h;\phi,l^k)d\iota_h - c^k_{ij\chi}(l^k;\phi)\int_0^{\mu^k_{xij}} x^k_j(\iota_x;\phi,l^k)d\iota_x, \tag{A.8}$$

where $\zeta^k_{ij\chi} = 1-\rho^k\alpha^k(1-\mu^k_{hij})\beta^k_{ij\chi} - \rho^k(1-\alpha^k)(1-\mu^k_{xij})(1-\beta^k_{ij\chi})$ as defined in equation (13) in the main text.

Using (A.8), we take the first-order condition with respect to $x^k_j(\iota_x;\phi,l^k)$ for $\iota_x \in [0,\mu^k_{xij}]$:

$$x^k_j(\iota_x;\phi,l^k) = (1-\alpha^k)\frac{\zeta^k_{ij\chi}}{\zeta^k_{ij}}\left(\frac{(1-\alpha)\rho\eta^k R_j(\phi)}{(X^k_j(\phi))^{\rho^k}}\right)^{\frac{1}{\zeta^k_{ij}}} \left(\exp\left[\int_0^{\mu^k_{hij}} \log h^k_j(\iota'_h;\phi,l^k)d\iota'_h\right]\right)^{\frac{\rho^k\alpha^k}{\zeta^k_{ij}}} \left(\exp\left[\int_0^{\mu^k_{xij}} \log x^k_j(\iota'_x;\phi,l^k)d\iota'_x\right]\right)^{\frac{\rho^k(1-\alpha^k)}{\zeta^k_{ij}}}$$
$$\times \left(\frac{\alpha^k\beta^k_{ij\chi}}{s_j}\right)^{\frac{\rho^k\alpha^k(1-\mu^k_{hij})}{\zeta^k_{ij}}} \left(\frac{(1-\alpha^k)\left(1-\beta^k_{ij\chi}\right)}{c^k_{ij\chi}(l^k;\phi)}\right)^{\frac{\rho^k(1-\alpha^k)(1-\mu^k_{xij})}{\zeta^k_{ij}}} \frac{1}{c^k_{ij\chi}(l^k;\phi)}.$$

As the right-hand side of this previous equation is identical for all $\iota_x \in [0,\mu^k_{xij}]$, this implies a common investment level in supplier contractible tasks, which we denote by $x^k_j(\iota_x;\phi,l^k) = x^k_{cj}(l^k;\phi)$ for all $\iota_x \in [0,\mu^k_{xij}]$. Substituting this symmetry back into the above first-order condition and simplifying, we have:

$$x^k_{cj}(l^k;\phi)^{\frac{1-\rho^k(1-\alpha^k\mu^k_{hij})}{\zeta^k_{ij}}} = (1-\alpha^k)\frac{\zeta^k_{ij\chi}}{\zeta^k_{ij}}\left(\frac{(1-\alpha)\rho\eta^k R_j(\phi)}{(X^k_j(\phi))^{\rho^k}}\right)^{\frac{1}{\zeta^k_{ij}}} \times \left(\exp\left[\int_0^{\mu^k_{hij}} \log h^k_j(\iota;\phi,l)d\iota\right]\right)^{\frac{\rho^k\alpha^k}{\zeta^k_{ij}}}$$
$$\times \left(\frac{\alpha^k\beta^k_{ij\chi}}{s_j}\right)^{\frac{\rho^k\alpha^k(1-\mu^k_{hij})}{\zeta^k_{ij}}} \left(\frac{(1-\alpha^k)\left(1-\beta^k_{ij\chi}\right)}{c^k_{ij\chi}(l^k;\phi)}\right)^{\frac{\rho^k(1-\alpha^k)(1-\mu^k_{xij})}{\zeta^k_{ij}}} \frac{1}{c^k_{ij\chi}(l^k;\phi)}. \tag{A.9}$$

Using (A.8) again, and taking the first-order condition with respect to $h^k_j(\iota_h;\phi,l^k)$ for $\iota_h \in [0,\mu^k_{hij}]$, we get:

$$h^k_j(\iota_h;\phi,l^k) = \alpha^k\frac{\zeta^k_{ij\chi}}{\zeta^k_{ij}}\left(\frac{(1-\alpha)\rho\eta^k R_j(\phi)}{(X^k_j(\phi))^{\rho^k}}\right)^{\frac{1}{\zeta^k_{ij}}} \left(\exp\left[\int_0^{\mu^k_{hij}} \log h^k_j(\iota_h;\phi,l^k)d\iota_h\right]\right)^{\frac{\rho^k\alpha^k}{\zeta^k_{ij}}} \left(\exp\left[\int_0^{\mu^k_{xij}} \log x^k_j(\iota_x;\phi,l^k)d\iota_x\right]\right)^{\frac{\rho^k(1-\alpha^k)}{\zeta^k_{ij}}}$$
$$\times \left(\frac{\alpha^k\beta^k_{ij\chi}}{s_j}\right)^{\frac{\rho^k\alpha^k(1-\mu^k_{hij})}{\zeta^k_{ij}}} \left(\frac{(1-\alpha^k)\left(1-\beta^k_{ij\chi}\right)}{c^k_{ij\chi}(l^k;\phi)}\right)^{\frac{\rho^k(1-\alpha^k)(1-\mu^k_{xij})}{\zeta^k_{ij}}} \frac{1}{s_j},$$

from which we can conclude that the investment levels that are specified for all contractible headquarter tasks are identical, i.e., $h_{cj}^k(l^k;\phi) = h_j^k(\iota_h;\phi,l^k)$ for all $\iota_h \in [0,\mu_{hij}^k]$ due to the symmetry across contractible headquarter tasks. Substituting this property back into the first-order condition and simplifying, we obtain:

$$h_{cj}^k(l^k;\phi)^{\frac{1-\rho^k(1-(1-\alpha^k)\mu_{xij}^k)}{\zeta_{ij}^k}} = \alpha^k \frac{\zeta_{ij\chi}^k}{\zeta_{ij}^k}\left(\frac{(1-\alpha)\rho\eta^k R_j(\phi)}{(X_j^k(\phi))^{\rho^k}}\right)^{\frac{1}{\zeta_{ij}^k}} \times \left(\exp\left[\int_0^{\mu_{xij}^k}\log x_j^k(\iota_x;\phi,l^k)d\iota_x\right]\right)^{\frac{\rho^k(1-\alpha^k)}{\zeta_{ij}^k}}$$
$$\times\left(\frac{\alpha^k\beta_{ij\chi}^k}{s_j}\right)^{\frac{\rho^k\alpha^k(1-\mu_{hij}^k)}{\zeta_{ij}^k}}\left(\frac{(1-\alpha^k)\left(1-\beta_{ij\chi}^k\right)}{c_{ij\chi}^k(l^k;\phi)}\right)^{\frac{\rho^k(1-\alpha^k)(1-\mu_{xij}^k)}{\zeta_{ij}^k}}\frac{1}{s_j}. \tag{A.10}$$

To solve out fully for $h_{cj}^k(l^k;\phi)$ and $x_{cj}^k(l^k;\phi)$, divide (A.10) by (A.9) to get:

$$h_{cj}^k(l^k;\phi) = \frac{\alpha^k}{1-\alpha^k}\frac{c_{ij\chi}^k(l^k;\phi)}{s_j}x_{cj}^k(l^k;\phi). \tag{A.11}$$

We substitute this last expression for $h_{cj}^k(l^k;\phi)$ into (A.10). Likewise, rearranging (A.11) implies an expression for $x_{cj}^k(l^k;\phi)$ in terms of $h_{cj}^k(l^k;\phi)$; we plug this expression for $x_{cj}^k(l^k;\phi)$ into (A.9). After some algebra, this yields:

$$x_{cj}^k(\phi,l)^{1-\rho^k} = \frac{(1-\alpha)\rho\eta^k R_j(\phi)}{(X_j^k(\phi))^{\rho^k}}\left(\frac{\alpha^k}{s_j}\right)^{\rho^k\alpha^k}\left(\frac{1-\alpha^k}{c_{ij\chi}^k(l^k;\phi)}\right)^{1-\rho^k\alpha^k}\left(\frac{\zeta_{ij\chi}^k}{\zeta_{ij}^k}\right)^{\zeta_{ij}^k}\left(\beta_{ij\chi}^k\right)^{\rho^k\alpha^k(1-\mu_{hij}^k)}\left(1-\beta_{ij\chi}^k\right)^{\rho^k(1-\alpha^k)(1-\mu_{xij}^k)}, \text{ and} \tag{A.12}$$

$$h_{cj}^k(\phi,l)^{1-\rho^k} = \frac{(1-\alpha)\rho\eta^k R_j(\phi)}{(X_j^k(\phi))^{\rho^k}}\left(\frac{\alpha^k}{s_j}\right)^{1-\rho^k(1-\alpha^k)}\left(\frac{1-\alpha^k}{c_{ij\chi}^k(l^k;\phi)}\right)^{\rho^k(1-\alpha^k)}\left(\frac{\zeta_{ij\chi}^k}{\zeta_{ij}^k}\right)^{\zeta_{ij}^k}\left(\beta_{ij\chi}^k\right)^{\rho^k\alpha^k(1-\mu_{hij}^k)}\left(1-\beta_{ij\chi}^k\right)^{\rho^k(1-\alpha^k)(1-\mu_{xij}^k)}. \tag{A.13}$$

The expressions for $h_{cj}^k(l^k;\phi)$ in (A.13) and for $x_{cj}^k(l^k;\phi)$ in (A.12) correspond to those reported in the main paper in the equation array in (12), bearing in mind the definition for $\Xi_{ij\chi}^k$ introduced in (15). Substituting from (A.13) and (A.12) into (A.7) and (A.6) then yields the expressions for $h_{nj}^k(l^k;\phi)$ and $x_{nj}^k(l^k;\phi)$ that are reported in (12) in the main text.

Plugging these expressions for $h_{cj}^k(l^k;\phi)$, $x_{cj}^k(l^k;\phi)$, $h_{nj}^k(l^k;\phi)$, and $x_{nj}^k(l^k;\phi)$ back into (A.8), the contribution to the firm's overall payoff that comes from its interaction with the supplier of l^k can be re-written as:

$$F_{ij\chi}^k(l^k;\phi) = \left(\frac{(1-\alpha)\rho\eta^k R_j(\phi)}{(X_j^k(\phi))^{\rho^k}}\right)^{\frac{1}{1-\rho^k}} \times \frac{1-\rho^k}{\rho^k}\left(\frac{\alpha^k}{s_j}\right)^{\frac{\rho^k\alpha^k}{1-\rho^k}}\left(\frac{1-\alpha^k}{c_{ij\chi}^k(l^k;\phi)}\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}$$
$$\times\left(\frac{\zeta_{ij\chi}^k}{\zeta_{ij}^k}\right)^{\frac{\zeta_{ij}^k}{1-\rho^k}}\left(\beta_{ij\chi}^k\right)^{\frac{\rho^k\alpha^k(1-\mu_{hij}^k)}{1-\rho^k}}\left(1-\beta_{ij\chi}^k\right)^{\frac{\rho^k(1-\alpha^k)(1-\mu_{xij}^k)}{1-\rho^k}}.$$

Given that $c_{ij\chi}^k(l^k;\phi) = w_i d_{ij}^k / Z_{ij\chi}^k(l^k;\phi)$, it is straightforward to see that the above expression is precisely equal to $F_{ij\chi}^k(l^k;\phi) = (\Xi_{ij\chi}^k Z_{ij\chi}^k(l^k;\phi))^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}$, as claimed at the start of Section 2.4 in the main text. This expresses the firm's payoff in terms of the fundamental parameters of the model and factor prices.

Properties of $h_{cj}^k(l^k;\phi)$, $x_{cj}^k(l^k;\phi)$, $h_{nj}^k(l^k;\phi)$, $x_{nj}^k(l^k;\phi)$: We show that the optimal task investment levels as reported in (12) are each increasing in the contractibility parameters μ_{hij}^k and μ_{xij}^k. Focus first on the contractible tasks, $h_{cj}^k(l^k;\phi)$ and $x_{cj}^k(l^k;\phi)$. These depend on μ_{hij}^k and μ_{xij}^k only through $\Xi_{ij\chi}^k$; from (15), this dependence operates solely through the $B_{ij\chi}^k$ term (bearing in mind that the decisions over optimal task levels for a given input variety are made taking the firm-level variables $R_j(\phi)$ and $X_j^k(\varphi)$ as given). That $B_{ij\chi}^k$ in (16) is increasing in both μ_{hij}^k and μ_{xij}^k is a property that we will derive shortly below in the proof of Lemma 1.

Turning to the investments for the noncontractible tasks, $h_{nj}^k(l^k;\phi)$ and $x_{nj}^k(l^k;\phi)$, it follows from (12) that it suffices to show that $\zeta_{ij}^k/\zeta_{ij\chi}^k$ is increasing in both μ_{hij}^k and μ_{xij}^k. Using the definitions in (13) and (14), and applying the quotient rule, one can show that:

$$\frac{d\zeta_{ij}^k/\zeta_{ij\chi}^k}{d\mu_{hij}^k} = \rho^k\alpha^k\left[\frac{(1-\rho^k(1-\alpha^k)(1-\mu_{xij}^k))(1-\beta_{ij\chi}^k)+\rho^k(1-\alpha^k)(1-\mu_{xij}^k)\beta_{ij\chi}^k}{(\zeta_{ij\chi}^k)^2}\right] > 0,$$

since ρ^k, α^k, $\beta_{ij\chi}^k$, $\mu_{xij}^k \in [0,1]$. The proof that $\zeta_{ij}^k/\zeta_{ij\chi}^k$ is increasing in μ_{xij}^k is analogous.

Sourcing shares: Denote the set of $2J$ possible sourcing modes by Ω, i.e., $\Omega = \{(i',\chi') : i' \in \{1,\ldots,J\}, \chi' \in \{V,O\}\}$. For a country-$j$ firm, the share of industry-k input varieties for which (i,χ) is the optimal sourcing mode, $\pi_{ij\chi}^k$, is evaluated explicitly as:

$$\pi_{ij\chi}^k = Pr\left(Z_{i'j\chi'}^k \leq \frac{\Xi_{ij\chi}^k}{\Xi_{i'j\chi'}^k}Z_{ij\chi}^k,\ \forall\ (i',\chi')\in\Omega\right) = \int_{z=0}^{\infty} Pr\left(Z_{ij\chi}^k = z, Z_{i'j\chi'}^k \leq \frac{\Xi_{ij\chi}^k}{\Xi_{i'j\chi'}^k}z,\ \forall\ (i',\chi')\neq(i,\chi)\right) dz.$$

We evaluate the above probability by differentiating the nested Fréchet joint distribution from equation (10) in the main paper with respect to $z_{ij\chi}^k$, setting $z_{ij\chi}^k = z$ and $z_{i'j\chi'}^k = (\Xi_{ij\chi}^k/\Xi_{i'j\chi'}^k)z$ for all $(i',\chi')\neq(i,\chi)$, and then integrating over all possible values of z. It follows that:

$$\begin{aligned}
\pi_{ij\chi}^k &= \int_{z=0}^{\infty}\exp\left\{-\sum_{i'=1}^{J}T_{i'}^k\left(\left(\frac{\Xi_{ij\chi}^k}{\Xi_{i'jV}^k}z\right)^{-\frac{\theta^k}{1-\lambda_{i'}}}+\left(\frac{\Xi_{ij\chi}^k}{\Xi_{i'jO}^k}z\right)^{-\frac{\theta^k}{1-\lambda_{i'}}}\right)^{1-\lambda_{i'}}\right\}\\
&\quad\times\left\{T_i^k\left(\left(\frac{\Xi_{ij\chi}^k}{\Xi_{ijV}^k}z\right)^{-\frac{\theta^k}{1-\lambda_i}}+\left(\frac{\Xi_{ij\chi}^k}{\Xi_{ijO}^k}z\right)^{-\frac{\theta^k}{1-\lambda_i}}\right)^{-\lambda_i}\right\}\times\theta^k z^{-\frac{\theta^k}{1-\lambda_i}-1}\ dz\\
&= \int_{z=0}^{\infty}\exp\left\{-\left(\Xi_{ij\chi}^k z\right)^{-\theta^k}\sum_{i'=1}^{J}T_{i'}^k\left(\left(\Xi_{i'jV}^k\right)^{\frac{\theta^k}{1-\lambda_{i'}}}+\left(\Xi_{i'jO}^k\right)^{\frac{\theta^k}{1-\lambda_{i'}}}\right)^{1-\lambda_{i'}}\right\}\\
&\quad\times\left\{T_i^k\left(\Xi_{ij\chi}^k\right)^{\frac{\theta^k\lambda_i}{1-\lambda_i}}\left(\left(\Xi_{ijV}^k\right)^{\frac{\theta^k}{1-\lambda_i}}+\left(\Xi_{ijO}^k\right)^{\frac{\theta^k}{1-\lambda_i}}\right)^{-\lambda_i}\right\}\times\theta^k z^{-\theta^k-1}\ dz\\
&= \underbrace{\frac{T_i^k\left(\left(\Xi_{ijV}^k\right)^{\frac{\theta^k}{1-\lambda_i}}+\left(\Xi_{ijO}^k\right)^{\frac{\theta^k}{1-\lambda_i}}\right)^{1-\lambda_i}}{\sum_{i'=1}^{J}T_{i'}^k\left(\left(\Xi_{i'jV}^k\right)^{\frac{\theta^k}{1-\lambda_{i'}}}+\left(\Xi_{i'jO}^k\right)^{\frac{\theta^k}{1-\lambda_{i'}}}\right)^{1-\lambda_{i'}}}}_{\equiv\ \pi_{ij}^k}\times\underbrace{\frac{\left(\Xi_{ij\chi}^k\right)^{\frac{\theta^k}{1-\lambda_i}}}{\left(\Xi_{ijV}^k\right)^{\frac{\theta^k}{1-\lambda_i}}+\left(\Xi_{ijO}^k\right)^{\frac{\theta^k}{1-\lambda_i}}}}_{\equiv\ \pi_{\chi|ij}^k}\\
&\quad\times\left[\exp\left\{-\left(\Xi_{ij\chi}^k z\right)^{-\theta^k}\sum_{i'=1}^{J}T_{i'}^k\left(\left(\Xi_{i'jV}^k\right)^{\frac{\theta^k}{1-\lambda_{i'}}}+\left(\Xi_{i'jO}^k\right)^{\frac{\theta^k}{1-\lambda_{i'}}}\right)^{1-\lambda_{i'}}\right\}\right]_{z=0}^{\infty}.
\end{aligned}$$

It is straightforward to see that the last term in square brackets is equal to 1, from which it follows that $\pi_{ij\chi}^k = \pi_{ij}\pi_{\chi|ij}$. Recalling the definitions of $\Xi_{ij\chi}^k$ from (15), $B_{ij\chi}^k$ from (16), and that $B_{ij}^k = ((B_{ijV}^k)^{\frac{\theta^k}{1-\lambda_i}}+(B_{ijO}^k)^{\frac{\theta^k}{1-\lambda_i}})^{\frac{1-\lambda_i}{\theta^k}}$, the above expressions for π_{ij}^k and $\pi_{\chi|ij}^k$ simplify to those in equation (19) in the main paper.

Properties of $B_{ij\chi}^k$ (Lemma 1): We show here that $B_{ij\chi}^k$ as defined in (16) is increasing in the contractibility of both headquarter and supplier tasks. Log-differentiating (16) with respect to μ_{hij}^k yields:

$$\begin{aligned}
\frac{\rho^k(1-\alpha^k)}{B_{ij\chi}^k}\frac{dB_{ij\chi}^k}{d\mu_{hij}^k} &= \frac{d\zeta_{ij}^k}{d\mu_{hij}^k}\log\frac{\zeta_{ij\chi}^k}{\zeta_{ij}^k}+\frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k}\frac{d\zeta_{ij\chi}^k}{d\mu_{hij}^k}-\frac{d\zeta_{ij}^k}{d\mu_{hij}^k}-\rho^k\alpha^k\log\beta_{ij\chi}^k\\
&= \rho^k\alpha^k\left(\frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k}\beta_{ij\chi}^k-\log\left(\frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k}\beta_{ij\chi}^k\right)-1\right).
\end{aligned}$$

Note from (13) and (14) that $\zeta_{ij}^k < \zeta_{ij\chi}^k$, since $\beta_{ij\chi}^k \in (0,1)$; it follows that $\frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k}\beta_{ij\chi}^k \in (0,1)$. It thus suffices to study the behavior of the function: $x - \log x - 1$ in the range $x \in (0,1)$. At $x = 0$, the value of this function tends to $+\infty$. Moreover, $\frac{d}{dx}(x - \log x - 1) = 1 - \frac{1}{x}$, which is negative for $x \in (0,1)$. As x increases from $x = 0$, $x - \log x - 1$ thus decreases monotonically until it reaches a value of 0 at $x = 1$. It follows that $x - \log x - 1 > 0$ for $x \in (0,1)$; this delivers the result that $B_{ij\chi}^k$ is increasing in μ_{hij}^k.

Similarly, log differentiating $B_{ij\chi}^k$ with respect to μ_{xij}^k yields:

$$\begin{aligned}\frac{\rho^k(1-\alpha^k)}{B_{ij\chi}^k}\frac{dB_{ij\chi}^k}{d\mu_{xij}^k} &= \frac{d\zeta_{ij}^k}{d\mu_{xij}^k}\log\frac{\zeta_{ij\chi}^k}{\zeta_{ij}^k} + \frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k}\frac{d\zeta_{ij\chi}^k}{d\mu_{xij}^k} - \frac{d\zeta_{ij}^k}{d\mu_{xij}^k} - \rho^k(1-\alpha^k)\log(1-\beta_{ij\chi}^k) \\ &= \rho^k(1-\alpha^k)\left(\frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k}(1-\beta_{ij\chi}^k) - \log\left(\frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k}(1-\beta_{ij\chi}^k)\right) - 1\right).\end{aligned}$$

Since $\frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k}(1-\beta_{ij\chi}^k) \in (0,1)$, and from the properties of the function $x - \log x - 1$ in the range $x \in (0,1)$ that have just been derived, it follows that $B_{ij\chi}^k$ is also increasing in μ_{xij}^k.

Recall that B_{ij}^k is a constant-elasticity aggregation of B_{ijV}^k and B_{ijO}^k (with exponent $\frac{\theta^k}{1-\lambda_i} > 0$). Since B_{ijV}^k and B_{ijO}^k are each increasing in both μ_{hij}^k and μ_{xij}^k, this implies B_{ij}^k is also increasing in both μ_{hij}^k and μ_{xij}^k.

Thus, B_{ijV}^k, B_{ijO}^k, and B_{ij}^k each achieve their maximum values when both μ_{hij}^k and μ_{xij}^k hit their upper bound of 1. From (13) and (14), $\zeta_{ijV}^k = \zeta_{ijO}^k = \zeta_{ij}^k = 1$ when $\mu_{hij}^k = \mu_{xij}^k = 1$; substituting further into (16) implies that the maximum value of both B_{ijV}^k and B_{ijO}^k is 1, while the maximum value of $(B_{ij}^k)^{\theta^k}$ is $2^{1-\lambda_i}$.

Behavior of $\pi_{V|ij}^k$ with respect to α^k (Lemma 2): The probability that a given input variety is purchased via an intrafirm transaction is given by equation (20) in the main paper. Observe that this expression for $\pi_{V|ij}^k$ is increasing in B_{ijV}^k/B_{ijO}^k, so $\pi_{V|ij}^k$ inherits the comparative static properties of B_{ijV}^k/B_{ijO}^k with respect to any model parameter. Using the definitions of B_{ijV}^k and B_{ijO}^k in (16), we have:

$$\frac{B_{ijV}^k}{B_{ijO}^k} = \left(\frac{\zeta_{ijV}^k}{\zeta_{ijO}^k}\right)^{\frac{\zeta_{ij}^k}{\rho^k(1-\alpha^k)}}\left(\frac{\beta_{ijV}^k}{\beta_{ijO}^k}\right)^{\frac{\alpha^k(1-\mu_{hij}^k)}{1-\alpha^k}}\left(\frac{1-\beta_{ijV}^k}{1-\beta_{ijO}^k}\right)^{1-\mu_{xij}^k},$$

so that:

$$\log\frac{B_{ijV}^k}{B_{ijO}^k} = \frac{\zeta_{ij}^k}{\rho^k(1-\alpha^k)}\log\frac{\zeta_{ijV}^k}{\zeta_{ijO}^k} + \frac{\alpha^k}{1-\alpha^k}(1-\mu_{hij}^k)\log\frac{\beta_{ijV}^k}{\beta_{ijO}^k} + (1-\mu_{xij}^k)\log\frac{1-\beta_{ijV}^k}{1-\beta_{ijO}^k}. \quad \text{(A.14)}$$

Differentiating (A.14) with respect to α^k, we get:

$$\frac{d}{d\alpha^k}\log\frac{B_{ijV}^k}{B_{ijO}^k} = \left(\frac{\zeta_{ij}^k}{\rho^k(1-\alpha^k)^2} + \frac{1}{\rho^k(1-\alpha^k)}\frac{d\zeta_{ij}^k}{d\alpha^k}\right)\log\frac{\zeta_{ijV}^k}{\zeta_{ijO}^k} + \frac{\zeta_{ij}^k}{\rho^k(1-\alpha^k)}\left(\frac{1}{\zeta_{ijV}^k}\frac{d\zeta_{ijV}^k}{d\alpha^k} - \frac{1}{\zeta_{ijO}^k}\frac{d\zeta_{ijO}^k}{d\alpha^k}\right) + \frac{1-\mu_{hij}^k}{(1-\alpha^k)^2}\log\frac{\beta_{ijV}^k}{\beta_{ijO}^k}.$$

From (14), $\frac{d\zeta_{ij}^k}{d\alpha^k} = \rho^k(\mu_{hij}^k - \mu_{xij}^k)$, so that: $\frac{\zeta_{ij}^k}{\rho^k(1-\alpha^k)^2} + \frac{1}{\rho^k(1-\alpha^k)}\frac{d\zeta_{ij}^k}{d\alpha^k} = \frac{1-\rho^k(1-\mu_{hij}^k)}{\rho^k(1-\alpha^k)^2}$. Also, note from (13) that: $\zeta_{ij\chi}^k = 1 - \rho^k(1-\mu_{xij}^k)(1-\beta_{ij\chi}^k) + \alpha^k\frac{d\zeta_{ij\chi}^k}{d\alpha^k}$, which implies: $\frac{1}{\zeta_{ij\chi}^k}\frac{d\zeta_{ij\chi}^k}{d\alpha^k} = \frac{1}{\alpha^k} - \frac{1-\rho^k(1-\mu_{xij}^k)(1-\beta_{ij\chi}^k)}{\alpha^k\zeta_{ij\chi}^k}$, for $\chi \in \{V, O\}$, and hence we get:

$$\frac{1}{\zeta_{ijV}^k}\frac{d\zeta_{ijV}^k}{d\alpha^k} - \frac{1}{\zeta_{ijO}^k}\frac{d\zeta_{ijO}^k}{d\alpha^k} = \frac{\rho^k(\beta_{ijV}^k - \beta_{ijO}^k)}{\zeta_{ijV}^k\zeta_{ijO}^k}\left[-(1-\mu_{hij}^k) - (1-\mu_{xij}^k) + \rho^k(1-\mu_{hij}^k)(1-\mu_{xij}^k)\right].$$

It follows that $\frac{d}{d\alpha^k}\log(B_{ijV}^k/B_{ijO}^k)$ is equal to $\frac{1}{\rho^k(1-\alpha^k)^2}\psi(\cdot)$, where:

$$\begin{aligned}\psi(\cdot) \equiv\ & (1-\rho^k(1-\mu_{hij}^k))\log\frac{\zeta_{ijV}^k}{\zeta_{ijO}^k} + \rho^k(1-\mu_{hij}^k)\log\frac{\beta_{ijV}^k}{\beta_{ijO}^k} \\ & + \frac{\rho^k(1-\alpha^k)(\beta_{ijV}^k - \beta_{ijO}^k)\zeta_{ij}^k}{\zeta_{ijV}^k\zeta_{ijO}^k}\left[-(1-\mu_{hij}^k) - (1-\mu_{xij}^k) + \rho^k(1-\mu_{hij}^k)(1-\mu_{xij}^k)\right],\end{aligned} \quad \text{(A.15)}$$

Viewing ψ as a function of β^k_{ijV}, note first that when $\beta^k_{ijV} = \beta^k_{ijO}$, we have $\psi = 0$. It then suffices to show that $\frac{d\psi}{d\beta^k_{ijV}} > 0$ for all $\beta^k_{ijV} \in (\beta^k_{ijO}, 1]$, in order to establish that $\psi > 0$ for all values of β^k_{ijV} over this range, and hence that $\frac{d}{d\alpha^k}\log(B^k_{ijV}/B^k_{ijO}) > 0$.

We therefore differentiate (A.15) with respect to β^k_{ijV} to obtain:

$$\frac{d\psi}{d\beta^k_{ijV}} = (1-\rho^k(1-\mu^k_{hij}))\frac{1}{\zeta^k_{ijV}}\frac{d\zeta^k_{ijV}}{d\beta^k_{ijV}} + \rho^k(1-\mu^k_{hij})\frac{1}{\beta^k_{ijV}}$$
$$+ \rho^k(1-\alpha^k)\frac{\zeta^k_{ij}}{\zeta^k_{ijO}}\left[-(1-\mu^k_{hij}) - (1-\mu^k_{xij}) + \rho^k(1-\mu^k_{hij})(1-\mu^k_{xij})\right]\frac{d}{d\beta^k_{ijV}}\frac{\beta^k_{ijV}-\beta^k_{ijO}}{\zeta^k_{ijV}}.$$

Using the quotient rule and some algebra, one can show that $\frac{d}{d\beta^k_{ijV}}\frac{\beta^k_{ijV}-\beta^k_{ijO}}{\zeta^k_{ijV}} = \frac{\zeta^k_{ijO}}{(\zeta^k_{ijV})^2}$, from which we get:

$$\frac{d\psi}{d\beta^k_{ijV}} = \frac{\rho^k}{(\zeta^k_{ijV})^2\beta^k_{ijV}}\Big[(1-\mu^k_{hij})\left(\zeta^k_{ijV}(1-\alpha^k\beta^k_{ijV}) - \zeta^k_{ij}(1-\alpha^k)\beta^k_{ijV}\right) + (1-\mu^k_{xij})(1-\alpha^k)\beta^k_{ijV}\left(\zeta^k_{ijV}-\zeta^k_{ij}\right)$$
$$+ \rho^k(1-\mu^k_{hij})(1-\mu^k_{xij})(1-\alpha^k)\left(\zeta^k_{ij}\beta^k_{ijV} - \zeta^k_{ijV}\right)\Big]. \tag{A.16}$$

Recall from the definitions of ζ^k_{ijV} and ζ^k_{ij} in (13) and (14) that with $\beta^k_{ijV} > 0$, we have: $\zeta^k_{ijV} > \zeta^k_{ij}$. Also, $\beta^k_{ijV} < 1$ implies: $1-\alpha^k\beta^k_{ijV} > (1-\alpha^k)\beta^k_{ijV}$. It follows that $\zeta^k_{ijV}(1-\alpha^k\beta^k_{ijV}) - \zeta^k_{ij}(1-\alpha^k)\beta^k_{ijV} > 0$ and $\zeta^k_{ijV} - \zeta^k_{ij} > 0$. Together with the fact that $\rho^k, (1-\mu^k_{hij}), (1-\mu^k_{xij}) \in (0,1)$, we have:

$$\frac{d\psi}{d\beta^k_{ijV}} > \frac{(\rho^k)^2}{(\zeta^k_{ijV})^2\beta^k_{ijV}}(1-\mu^k_{hij})(1-\mu^k_{xij})$$
$$\times\Big[\left(\zeta^k_{ijV}(1-\alpha^k\beta^k_{ijV}) - \zeta^k_{ij}(1-\alpha^k)\beta^k_{ijV}\right) + (1-\alpha^k)\beta^k_{ijV}\left(\zeta^k_{ijV}-\zeta^k_{ij}\right) + (1-\alpha^k)\left(\zeta^k_{ij}\beta^k_{ijV} - \zeta^k_{ijV}\right)\Big]$$
$$= \frac{(\rho^k)^2}{(\zeta^k_{ijV})^2\beta^k_{ijV}}(1-\mu^k_{hij})(1-\mu^k_{xij}) \times \Big[\zeta^k_{ijV}\left(\alpha^k(1-\beta^k_{ijV}) + (1-\alpha^k)\beta^k_{ijV}\right) - \zeta^k_{ij}(1-\alpha^k)\beta^k_{ijV}\Big].$$

That this last expression is positive follows from the fact that $\zeta^k_{ijV} > \zeta^k_{ij}$.

Behavior of $\pi^k_{V|ij}$ with respect to μ^k_{hij} and μ^k_{xij} (Lemma 2): Differentiating (A.14) with respect to μ^k_{hij}:

$$\frac{d}{d\mu^k_{hij}}\log\frac{B^k_{ijV}}{B^k_{ijO}} = \frac{1}{\rho^k(1-\alpha^k)}\left[\frac{d\zeta^k_{ij}}{d\mu^k_{hij}}\log\frac{\zeta^k_{ijV}}{\zeta^k_{ijO}} + \zeta^k_{ij}\left(\frac{1}{\zeta^k_{ijV}}\frac{d\zeta^k_{ijV}}{d\mu^k_{hij}} - \frac{1}{\zeta^k_{ijO}}\frac{d\zeta^k_{ijO}}{d\mu^k_{hij}}\right) - \rho^k\alpha^k\log\frac{\beta^k_{ijV}}{\beta^k_{ijO}}\right]$$
$$= \frac{\alpha^k}{1-\alpha^k}\left[\log\frac{\zeta^k_{ij}}{\zeta^k_{ijO}}\beta^k_{ijO} - \frac{\zeta^k_{ij}}{\zeta^k_{ijO}}\beta^k_{ijO} - \log\frac{\zeta^k_{ij}}{\zeta^k_{ijV}}\beta^k_{ijV} + \frac{\zeta^k_{ij}}{\zeta^k_{ijV}}\beta^k_{ijV}\right].$$

The function $\log x - x$ is increasing in x for all $x \in (0,1)$, since $\frac{d}{dx}(\log x - x) = \frac{1}{x} - 1 > 0$ over this range of values for x. Moreover, we have seen that $\zeta^k_{ij} < \zeta^k_{ij\chi}$, while $\beta^k_{ij\chi} \in (0,1)$, so that $\frac{\zeta^k_{ij}}{\zeta^k_{ij\chi}}\beta^k_{ij\chi} \in (0,1)$ for both $\chi = V$ and $\chi = O$. Note that: $\frac{\zeta^k_{ij}}{\zeta^k_{ijV}}\beta^k_{ijV} - \frac{\zeta^k_{ij}}{\zeta^k_{ijO}}\beta^k_{ijO} = \frac{\zeta^k_{ij}}{\zeta^k_{ijV}\zeta^k_{ijO}}(\beta^k_{ijV} - \beta^k_{ijO})(1-\rho^k(1-\alpha^k)(1-\mu^k_{xij})) > 0$, since $\beta^k_{ijV} > \beta^k_{ijO}$. The behavior of the $\log x - x$ function over $x \in (0,1)$ then implies that: $\log\frac{\zeta^k_{ij}}{\zeta^k_{ijO}}\beta^k_{ijO} - \frac{\zeta^k_{ij}}{\zeta^k_{ijO}}\beta^k_{ijO} < \log\frac{\zeta^k_{ij}}{\zeta^k_{ijV}}\beta^k_{ijV} - \frac{\zeta^k_{ij}}{\zeta^k_{ijV}}\beta^k_{ijV}$. Hence, $\frac{d}{d\mu^k_{hij}}\log(B^k_{ijV}/B^k_{ijO}) < 0$, and $\pi^k_{V|ij}$ is decreasing in μ^k_{hij}.

Differentiating (A.14) now with respect to μ^k_{xij}, we have an analogous expression:

$$\frac{d}{d\mu^k_{xij}}\log\frac{B^k_{ijV}}{B^k_{ijO}} = \log\frac{\zeta^k_{ijV}}{\zeta^k_{ijO}} + \frac{\zeta^k_{ij}}{\zeta^k_{ijV}}(1-\beta^k_{ijV}) - \frac{\zeta^k_{ij}}{\zeta^k_{ijO}}(1-\beta^k_{ijO}) - \log\frac{1-\beta^k_{ijV}}{1-\beta^k_{ijO}}$$
$$= \log\frac{\zeta^k_{ij}}{\zeta^k_{ijO}}(1-\beta^k_{ijO}) - \frac{\zeta^k_{ij}}{\zeta^k_{ijO}}(1-\beta^k_{ijO}) - \log\frac{\zeta^k_{ij}}{\zeta^k_{ijV}}(1-\beta^k_{ijV}) + \frac{\zeta^k_{ij}}{\zeta^k_{ijV}}(1-\beta^k_{ijV}).$$

Note that $\frac{\zeta^k_{ij}}{\zeta^k_{ij\chi}}(1-\beta^k_{ij\chi}) \in (0,1)$ for both $\chi = V$ and $\chi = O$. Moreover: $\frac{\zeta^k_{ij}}{\zeta^k_{ijV}}(1-\beta^k_{ijV}) - \frac{\zeta^k_{ij}}{\zeta^k_{ijO}}(1-\beta^k_{ijO}) = \frac{\zeta^k_{ij}}{\zeta^k_{ijV}\zeta^k_{ijO}}(\beta^k_{ijO} - \beta^k_{ijV})(1-\rho^k\alpha^k(1-\mu^k_{hij})) < 0$. Together with the fact that $\log x - x$ is increasing over the interval $(0,1)$, we have: $\log \frac{\zeta^k_{ij}}{\zeta^k_{ijO}}(1-\beta^k_{ijO}) - \frac{\zeta^k_{ij}}{\zeta^k_{ijO}}(1-\beta^k_{ijO}) > \log \frac{\zeta^k_{ij}}{\zeta^k_{ijV}}(1-\beta^k_{ijV}) - \frac{\zeta^k_{ij}}{\zeta^k_{ijV}}(1-\beta^k_{ijV})$. Hence, $\frac{d}{d\mu^k_{xij}} \log(B^k_{ijV}/B^k_{ijO}) > 0$, and $\pi^k_{V|ij}$ is increasing in μ^k_{xij}.

Behavior of $\pi^k_{V|ij}$ with respect to β^k_{ijO} (Lemma 2): Recall that $\beta^k_{ijV} = \delta^k_{ij} + \beta^k_{ijO}(1-\delta^k_{ij})$. First, note that: $1-\beta^k_{ijV} = (1-\beta^k_{ijO})(1-\delta^k_{ij})$. Substituting this into (A.14) and differentiating with respect to β^k_{ijO}, we get:

$$\frac{d}{d\beta^k_{ijO}} \log \frac{B^k_{ijV}}{B^k_{ijO}} = \frac{\zeta^k_{ij}}{\rho^k(1-\alpha^k)} \left(\frac{1}{\zeta^k_{ijV}} \frac{d\zeta^k_{ijV}}{d\beta^k_{ijO}} - \frac{1}{\zeta^k_{ijO}} \frac{d\zeta^k_{ijO}}{d\beta^k_{ijO}} \right) - \frac{\alpha^k}{1-\alpha^k}(1-\mu^k_{hij}) \frac{\delta^k_{ij}}{\beta^k_{ijO}\beta^k_{ijV}}.$$

Next, from the definition in (13), one has: $\zeta^k_{ijO} = 1 - \rho^k(1-\alpha^k)(1-\mu^k_{xij}) + \beta^k_{ijO}\frac{d\zeta^k_{ijO}}{d\beta^k_{ijO}}$, from which we have: $\frac{d\zeta^k_{ijO}}{d\beta^k_{ijO}} = \frac{1}{\beta^k_{ijO}}\left(\zeta^k_{ijO} - 1 + \rho^k(1-\alpha^k)(1-\mu^k_{xij})\right)$. Moreover: $\frac{d\zeta^k_{ijV}}{d\beta^k_{ijO}} = \frac{d\zeta^k_{ijV}}{d\beta^k_{ijV}}\frac{d\beta^k_{ijV}}{d\beta^k_{ijO}} = (1-\delta^k_{ij})\frac{d\zeta^k_{ijO}}{d\beta^k_{ijO}}$. Using these expressions and with some algebra, one can show that: $\frac{1}{\zeta^k_{ijV}}\frac{d\zeta^k_{ijV}}{d\beta^k_{ijO}} - \frac{1}{\zeta^k_{ijO}}\frac{d\zeta^k_{ijO}}{d\beta^k_{ijO}} = -(1-\rho^k\alpha^k(1-\mu^k_{hij}))\frac{\delta^k_{ij}}{\zeta^k_{ijV}\zeta^k_{ijO}}\frac{d\zeta^k_{ijO}}{d\beta^k_{ijO}}$. The derivative of $\log(B^k_{ijV}/B^k_{ijO})$ that is of interest is then given by:

$$\frac{d}{d\beta^k_{ijO}} \log \frac{B^k_{ijV}}{B^k_{ijO}} = \frac{\delta^k_{ij}\alpha^k(1-\mu^k_{hij})}{1-\alpha^k} \left[\frac{\zeta^k_{ij}}{\zeta^k_{ijV}\zeta^k_{ijO}}(1-\rho^k\alpha^k(1-\mu^k_{hij})) \left(1 - \frac{(1-\alpha^k)(1-\mu^k_{xij})}{\alpha^k(1-\mu^k_{hij})}\right) - \frac{1}{\beta^k_{ijV}\beta^k_{ijO}} \right]. \tag{A.17}$$

Consider first the case where: $\alpha^k(1-\mu^k_{hij}) - (1-\alpha^k)(1-\mu^k_{xij}) \le 0$, so that $1 - \frac{(1-\alpha^k)(1-\mu^k_{xij})}{\alpha^k(1-\mu^k_{hij})} \le 0$. Since ζ^k_{ij}, ζ^k_{ijV}, ζ^k_{ijO}, and $1-\rho^k\alpha^k(1-\mu^k_{hij})$ are all positive, (A.17) immediately implies that: $\frac{d}{d\beta^k_{ijO}}\log(B^k_{ijV}/B^k_{ijO}) < 0$.

Consider then the converse case: $\alpha^k(1-\mu^k_{hij}) - (1-\alpha^k)(1-\mu^k_{xij}) > 0$, under which we have: $1 - \frac{(1-\alpha^k)(1-\mu^k_{xij})}{\alpha^k(1-\mu^k_{hij})} \in (0,1)$. One can directly verify that: $\zeta^k_{ijO} - (1-\rho^k\alpha^k(1-\mu^k_{hij})) = \rho^k(\alpha^k(1-\mu^k_{hij}) - (1-\alpha^k)(1-\mu^k_{xij}))(1-\beta^k_{ijO}) > 0$; this means we have: $\frac{1-\rho^k\alpha^k(1-\mu^k_{hij})}{\zeta^k_{ijO}} \in (0,1)$. Last but not least, recall that $\frac{\zeta^k_{ij}}{\zeta^k_{ijV}} \in (0,1)$. We therefore have: $\frac{\zeta^k_{ij}}{\zeta^k_{ijV}\zeta^k_{ijO}}(1-\rho^k\alpha^k(1-\mu^k_{hij}))(1 - \frac{(1-\alpha^k)(1-\mu^k_{xij})}{\alpha^k(1-\mu^k_{hij})}) \in (0,1)$, whereas $-\frac{1}{\beta^k_{ijV}\beta^k_{ijO}} < -1$. It follows from (A.17) that: $\frac{d}{d\beta^k_{ijO}}\log(B^k_{ijV}/B^k_{ijO}) < 0$ also in this second case.

Behavior of $\pi^k_{V|ij}$ with respect to δ^k_{ij} (Lemma 2): Substituting $1-\beta^k_{ijV} = (1-\beta^k_{ijO})(1-\delta^k_{ij})$ into (A.14), and then differentiating with respect to δ^k_{ij}, we obtain:

$$\begin{aligned}\frac{d}{d\delta^k_{ij}} \log \frac{B^k_{ijV}}{B^k_{ijO}} &= \frac{1}{\rho^k(1-\alpha^k)} \frac{\zeta^k_{ij}}{\zeta^k_{ijV}} \frac{d\zeta^k_{ijV}}{d\delta^k_{ij}} + \frac{\alpha^k}{1-\alpha^k}(1-\mu^k_{hij}) \frac{1-\beta^k_{ijO}}{\beta^k_{ijV}} - (1-\mu^k_{xij})\frac{1}{1-\delta^k_{ij}} \\ &= \frac{1}{1-\alpha^k}\left[\frac{\zeta^k_{ij}}{\zeta^k_{ijV}}(1-\beta^k_{ijO})\left(-\alpha^k(1-\mu^k_{hij}) + (1-\alpha^k)(1-\mu^k_{xij})\right) + \alpha^k(1-\mu^k_{hij})\frac{1-\beta^k_{ijO}}{\beta^k_{ijV}} - \frac{(1-\alpha^k)(1-\mu^k_{xij})}{1-\delta^k_{ij}} \right].\end{aligned} \tag{A.18}$$

Let Ψ denote the expression in square brackets in this last line above. We view Ψ as a function of δ^k_{ij}, and seek to understand its behavior over the interval $\delta^k_{ij} \in (0,1)$ in order to sign how $\log(B^k_{ijV}/B^k_{ijO})$ and hence how $\pi^k_{V|ij}$ varies with δ^k_{ij}. We have:

$$\frac{d\Psi}{d\delta^k_{ij}} = -\rho^k \frac{\zeta^k_{ij}}{(\zeta^k_{ijV})^2}(1-\beta^k_{ijO})^2\left(-\alpha^k(1-\mu^k_{hij}) + (1-\alpha^k)(1-\mu^k_{xij})\right)^2 - \alpha^k(1-\mu^k_{hij})\left(\frac{1-\beta^k_{ijO}}{\delta^k_{ij} + (1-\delta^k_{ij})\beta^k_{ijO}}\right)^2 - \frac{(1-\alpha^k)(1-\mu^k_{xij})}{(1-\delta^k_{ij})^2}. \tag{A.19}$$

The above is clearly negative, so Ψ is a monotonic decreasing function of δ^k_{ij} over the interval $\delta^k_{ij} \in (0,1)$, that moreover tends to $-\infty$ as $\delta^k_{ij} \longrightarrow 1$. It suffices therefore to examine the sign of Ψ when evaluated at $\delta^k_{ij} = 0$: If $\Psi < 0$ at $\delta^k_{ij} = 0$, then we can conclude that $\Psi < 0$ and hence $\frac{d}{d\delta^k_{ij}}\log(B^k_{ijV}/B^k_{ijO}) < 0$ for all $\delta^k_{ij} \in (0,1)$. On the other hand, if $\Psi > 0$ at $\delta^k_{ij} = 0$, then there exists a unique cutoff value of $\bar{\delta}^k_{ij} \in (0,1)$ such that $\frac{d}{d\delta^k_{ij}}\log(B^k_{ijV}/B^k_{ijO}) > 0$ when $\delta^k_{ij} \in (0, \bar{\delta}^k_{ij})$, while $\frac{d}{d\delta^k_{ij}}\log(B^k_{ijV}/B^k_{ijO}) < 0$ when $\delta^k_{ij} \in (\bar{\delta}^k_{ij}, 1)$. (As will be evident from the discussion below, $\bar{\delta}_{ij}^k$ depends on the values of the other deep parameters of the model, including α^k.)

Setting $\delta_{ij}^k = 0$ in (A.18), and after some algebraic simplification, we have:

$$\Psi(0) = \frac{1}{\beta_{ijO}^k \zeta_{ijO}^k} \left[\alpha^k (1-\mu_{hij}^k)(1-\beta_{ijO}^k)^2 - (1-\alpha^k)(1-\mu_{xij}^k)(\beta_{ijO}^k)^2 + \rho^k \alpha^k (1-\alpha^k)(1-\mu_{hij}^k)(1-\mu_{xij}^k)(2\beta_{ijO}^k - 1) \right].$$

Thus, $\Psi(0) > 0$ if and only if:

$$\frac{\alpha^k (1-\mu_{hij}^k) \left(1 - \rho^k (1-\alpha^k)(1-\mu_{xij}^k)\right) (1-\beta_{ijO}^k)^2}{(1-\alpha^k)(1-\mu_{xij}^k)(1-\rho^k \alpha^k (1-\mu_{hij}^k))(\beta_{ijO}^k)^2} \;>\; 1.$$

The left-hand side of this last inequality takes on non-negative values: It equals 0 at $\alpha^k = 0$, while tending to $+\infty$ at $\alpha^k = 1$. It is also straightforward to see that it increases monotonically in α^k, since the numerator is an increasing function in α^k while the denominator is a decreasing function in α^k. The intermediate value theorem then implies there exists a unique $\bar{\alpha}^k$ such that $\Psi(0) > 0$ when $\alpha^k > \bar{\alpha}^k$, while $\Psi(0) < 0$ when $\alpha^k < \bar{\alpha}^k$. When $\alpha^k < \bar{\alpha}^k$, $\log(B_{ijV}^k / B_{ijO}^k)$ and hence $\pi_{V|ij}^k$ are decreasing in δ_{ij}^k for all $\delta_{ij}^k \in (0,1)$. On the other hand, when $\alpha^k > \bar{\alpha}^k$, $\pi_{V|ij}^k$ is increasing in δ_{ij}^k over the interval $(0, \bar{\delta}_{ij}^k)$, while decreasing in δ_{ij}^k over the interval $(\bar{\delta}_{ij}^k, 1)$. Summarizing the result across the two α^k cases, the propensity to integrate falls in δ_{ij}^k for δ_{ij}^k sufficiently large.

A.2 Proofs from Section 3

Composite industry-k input: Based on the definition of $X_j^k(\phi)$ in (3), we have:

$$\begin{aligned} X_j^k(\phi)^{\rho^k} \;&=\; \mathbb{E}_{l^k}[\tilde{x}_j^k(l^k;\phi)^{\rho^k}] \;= \sum_{i=1}^{J} \sum_{\chi \in \{V,O\}} \pi_{ij}^k \pi_{\chi|ij}^k \times \mathbb{E}_{l^k} \left[\tilde{x}_j^k(l^k;\phi)^{\rho^k} | (i,\chi) = \arg\max_{(i',\chi')} \; \Xi_{i'j\chi'}^k Z_{i'j\chi'}^k(l^k;\phi) \right] \\ &= \sum_{i=1}^{J} \sum_{\chi \in \{V,O\}} \mathbb{E}_{l^k} \left[\tilde{x}_j^k(l^k;\phi)^{\rho^k} \;\&\; (i,\chi) = \arg\max_{(i',\chi')} \; \Xi_{i'j\chi'}^k Z_{i'j\chi'}^k(l^k;\phi) \right]. \end{aligned}$$

We obtain an expression for $\tilde{x}_j^k(l^k;\phi)$, by substituting in the optimal task investment levels $h_{cj}^k(l^k;\phi)$, $h_{nj}^k(l^k;\phi)$, $x_{cj}^k(l^k;\phi)$, and $x_{nj}^k(l^k;\phi)$ from (12) into the definitions of $h_j^k(l^k;\phi)$ in (5) and $x_j^k(l^k;\phi)$ in (6), and in turn into the definition of $\tilde{x}_j^k(l^k;\phi)$ in (4). With some simplification, this yields:

$$\begin{aligned} X_j^k(\phi)^{\rho^k} = \sum_{i=1}^{J} \sum_{\chi \in \{V,O\}} & \left(\frac{\alpha^k (\beta_{ij\chi}^k)^{1-\mu_{hij}^k}}{s_j} \right)^{\rho^k \alpha^k} \left(\frac{(1-\alpha^k)(1-\beta_{ij\chi}^k)^{1-\mu_{xij}^k}}{d_{ij}^k w_i} \right)^{\rho^k (1-\alpha^k)} \left(\frac{\rho^k}{1-\rho^k} \right)^{\rho^k} \\ & \times \left(\frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k} \right)^{1-\zeta_{ij}^k} (\Xi_{ij\chi}^k)^{\frac{(\rho^k)^2 (1-\alpha^k)}{1-\rho^k}} \mathbb{E}_{l^k}[\tilde{Z}_{ij\chi}^k(l^k;\phi)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}], \end{aligned} \tag{A.20}$$

where $\tilde{Z}_{ij\chi}^k(l^k;\phi)$ denotes the value of the supplier draw $Z_{ij\chi}^k(l^k;\phi)$ when (i,χ) is in fact the optimal sourcing mode that solves the decision problem in (17).

Define: $\bar{z}_{ij\chi}^k \equiv \mathbb{E}_{l^k}[\tilde{Z}_{ij\chi}(l^k;\phi)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}]$, since this expression will appear repeatedly. We have:

$$\begin{aligned} \bar{z}_{ij\chi}^k = \int_{z=0}^{\infty} z^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} \; \exp\left\{ - \left(\Xi_{ij\chi}^k z\right)^{-\theta^k} \sum_{i'=1}^{J} T_{i'}^k \left(\left(\Xi_{i'jV}^k\right)^{\frac{\theta^k}{1-\lambda_{i'}}} + \left(\Xi_{i'jO}^k\right)^{\frac{\theta^k}{1-\lambda_{i'}}} \right)^{1-\lambda_{i'}} \right\} \\ \times \left\{ T_i^k \left(\Xi_{ij\chi}^k\right)^{\frac{\theta^k \lambda_i}{1-\lambda_i}} \left(\left(\Xi_{ijV}^k\right)^{\frac{\theta^k}{1-\lambda_i}} + \left(\Xi_{ijO}^k\right)^{\frac{\theta^k}{1-\lambda_i}} \right)^{-\lambda_i} \right\} \times \theta^k z^{-\theta^k - 1} \; dz. \end{aligned}$$

Implementing the change of variables: $y = \left(\Xi_{ij\chi}^k z\right)^{-\theta^k} \upsilon_j^k$, with $\upsilon_j^k \equiv \sum_{i'=1}^{J} T_{i'}^k \left(\left(\Xi_{i'jV}^k\right)^{\frac{\theta^k}{1-\lambda_{i'}}} + \left(\Xi_{i'jO}^k\right)^{\frac{\theta^k}{1-\lambda_{i'}}} \right)^{1-\lambda_{i'}}$:

$$
\begin{aligned}
\bar{z}^k_{ij\chi} &= \int_{z=0}^{\infty} \exp\{-y\}\, T^k_i \left(\Xi^k_{ij\chi}\right)^{\frac{\theta^k\lambda_i}{1-\lambda_i}} \left(\left(\Xi^k_{ijV}\right)^{\frac{\theta^k}{1-\lambda_i}} + \left(\Xi^k_{ijO}\right)^{\frac{\theta^k}{1-\lambda_i}}\right)^{-\lambda_i} \theta^k z^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}-\theta^k-1} dz \\
&= \int_{y=0}^{\infty} \exp\{-y\}\, T^k_i \left(\Xi^k_{ij\chi}\right)^{\frac{\theta^k\lambda_i}{1-\lambda_i}} \left(\left(\Xi^k_{ijV}\right)^{\frac{\theta^k}{1-\lambda_i}} + \left(\Xi^k_{ijO}\right)^{\frac{\theta^k}{1-\lambda_i}}\right)^{-\lambda_i} \left(\frac{y}{\upsilon^k_j}\right)^{-\frac{1}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} \left(\Xi^k_{ij\chi}\right)^{\theta^k-\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} \frac{1}{\upsilon^k_j}\, dy \\
&= (\Xi^k_{ij\chi})^{\frac{\theta^k}{1-\lambda_i}-\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} \times \frac{T^k_i\left((\Xi^k_{ijV})^{\frac{\theta^k}{1-\lambda_i}} + (\Xi^k_{ijO})^{\frac{\theta^k}{1-\lambda_i}}\right)^{-\lambda_i}}{(\upsilon^k_j)^{1-\frac{1}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}} \times \int_{y=0}^{\infty} \exp\{-y\}\, y^{-\frac{1}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}\, dy \\
&= \bar{\Gamma}^k \times \left(\frac{(\Xi^k_{ij\chi})^{\frac{\theta^k}{1-\lambda_i}}}{(\Xi^k_{ijV})^{\frac{\theta^k}{1-\lambda_i}} + (\Xi^k_{ijO})^{\frac{\theta^k}{1-\lambda_i}}}\right)^{1-\frac{1-\lambda_i}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} \left(\frac{T^k_i\left((\Xi^k_{ijV})^{\frac{\theta^k}{1-\lambda_i}} + (\Xi^k_{ijO})^{\frac{\theta^k}{1-\lambda_i}}\right)^{1-\lambda_i}}{\upsilon^k_j}\right)^{1-\frac{1}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} \left(T^k_i\right)^{\frac{1}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k}},
\end{aligned}
$$

where recall that the constant, $\bar{\Gamma}^k$, is the value of the Gamma function, $\Gamma(t) \equiv \int_{y=0}^{\infty} \exp\{-y\}\, y^{t-1}\, dy$, when this is evaluated at $t = 1 - \frac{1}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k}$. (We require that: $1 - \frac{1}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k} > 0$, in order for $\bar{\Gamma}^k$ to be well-defined.)

Using the expressions for the sourcing shares π^k_{ij} and $\pi^k_{\chi|ij}$ from Section A.1, $\bar{z}^k_{ij\chi}$ simplifies to:

$$
\begin{aligned}
\bar{z}^k_{ij\chi} &= \bar{\Gamma}^k \times \left(\pi^k_{ij}\right)^{1-\frac{1}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} \left(\pi^k_{\chi|ij}\right)^{1-\frac{1-\lambda_i}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} \left(T^k_i\right)^{\frac{1}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}. \\
&= \bar{\Gamma}^k \times \pi^k_{ij}\pi^k_{\chi|ij} \times \left(\frac{(B^k_{ij\chi})^{\frac{\theta^k}{1-\lambda_i}}}{(B^k_{ijV})^{\frac{\theta^k}{1-\lambda_i}} + (B^k_{ijO})^{\frac{\theta^k}{1-\lambda_i}}}\right)^{-\frac{1-\lambda_i}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} \left(\frac{T^k_i(d^k_{ij}w_i)^{-\theta^k}(B^k_{ij})^{\theta^k}}{\Phi^k_j}\right)^{-\frac{1}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} \left(T^k_i\right)^{\frac{1}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} \\
&= \bar{\Gamma}^k \times \pi^k_{ij}\pi^k_{\chi|ij} \left(\Phi^k_j\right)^{\frac{1}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} \left(\frac{d^k_{ij}w_i}{B^k_{ij\chi}}\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}, \qquad \text{(A.21)}
\end{aligned}
$$

where $\Phi^k_j = \sum_{i'=1}^{J} T^k_{i'}(d^k_{i'j}w_{i'})^{-\theta^k}(B^k_{i'j})^{\theta^k}$ is the sourcing capability of country-j firms over industry-k inputs, from equation (22) in the main paper.

We next substitute this expression for $\bar{z}^k_{ij\chi}$ from (A.21) into (A.20). At the same time, we substitute in for $\Xi^k_{ij\chi}$ from (15) and for $B^k_{ij\chi}$ from (16). After some extensive algebra, one arrives at the expression for $X^k_j(\phi)$ reported in equation (21) in the main paper:

$$
X^k_j(\phi) = (1-\alpha)\rho\eta^k R_j(\phi)(\alpha^k/s_j)^{\alpha^k}(1-\alpha^k)^{1-\alpha^k}(\bar{\Gamma}^k)^{\frac{1-\rho^k}{\rho^k}}(\Phi^k_j)^{\frac{1-\alpha^k}{\theta^k}}(\Upsilon^k_j)^{\frac{1-\rho^k}{\rho^k}},
$$

where: $\Upsilon^k_j = \sum_{i=1}^{J}\sum_{\chi\in\{V,O\}} \frac{\zeta^k_{ij}}{\zeta^k_{ij\chi}}\pi^k_{ij}\pi^k_{\chi|ij}$ in the under-investment friction term from equation (21).

Payoff of the final-good firm: We compute an expression for the (pre-assembly) payoff for the final-good firm, $F_j(\phi)$; this will then allow us to derive the optimal amount of assembly labor chosen by the firm.

Taking into account all factor costs incurred and ex-ante transfers received in its bilateral interactions with all suppliers, we have:

$$
\begin{aligned}
F_j(\phi) &= R_j(\phi) - \sum_{k=1}^{K}\int_{l^k=0}^{1}\left[s_j\int_0^1 h^k_j(\iota_h;\phi,l^k)d\iota_h + c^k_{ij\chi}(l^k;\phi)\int_0^1 x^k_j(\iota_x;\phi,l^k)d\iota_x\right]dl^k - w_jL_j(\phi) \\
&= R_j(\phi) - \sum_{k=1}^{K}\int_{l^k=0}^{1} s_j\left[\mu^k_{hij}h^k_{cj}(l^k;\phi) + (1-\mu^k_{hij})h^k_{nj}(l^k;\phi)\right]dl^k \\
&\qquad - \sum_{k=1}^{K}\int_{l^k=0}^{1} c^k_{ij\chi}(l^k;\phi)\left[\mu^k_{xij}x^k_{cj}(l^k;\phi) + (1-\mu^k_{xij})x^k_{nj}(l^k;\phi)\right]dl^k - w_jL_j(\phi). \qquad \text{(A.22)}
\end{aligned}
$$

Using the expressions for $h^k_{cj}(l^k;\phi)$ and $h^k_{nj}(l^k;\phi)$ from (12), the total factor costs incurred in the provision of headquarter services can be written after some simplification as:

$$\sum_{k=1}^{K}\int_0^1 s_j\left[\mu^k_{hij}h^k_{cj}(l^k;\phi)+(1-\mu^k_{hij})h^k_{nj}(l^k;\phi)\right]dl^k = \sum_{k=1}^{K}\frac{\rho^k}{1-\rho^k}\alpha^k\mathbb{E}_{l^k}\left[\left(\mu^k_{hij}+(1-\mu^k_{hij})\beta^k_{ij\chi}\left(\frac{\zeta^k_{ij}}{\zeta^k_{ij\chi}}\right)\right)\left(\Xi^k_{ij\chi}\tilde{Z}_{ij\chi}(l^k;\phi)\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}\right]$$
$$= \sum_{k=1}^{K}\frac{\rho^k}{1-\rho^k}\alpha^k\sum_{i=1}^{J}\sum_{\chi\in\{V,O\}}\left(\mu^k_{hij}+(1-\mu^k_{hij})\beta^k_{ij\chi}\left(\frac{\zeta^k_{ij}}{\zeta^k_{ij\chi}}\right)\right)\left(\Xi^k_{ij\chi}\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}\bar{z}^k_{ij\chi},$$

where recall that $\tilde{Z}_{ij\chi}(l^k;\phi)$ denotes the supplier draw when (i,χ) is in fact the optimal sourcing mode, and $\bar{z}^k_{ij\chi}\equiv\mathbb{E}_{l^k}[\tilde{Z}_{ij\chi}(l^k;\phi)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}]$. On the other hand, using the expressions for $x^k_{cj}(l^k;\phi)$ and $x^k_{nj}(l^k;\phi)$ from (12), and working through similar algebraic steps, the total factor costs incurred in supplier tasks can be written as:

$$\sum_{k=1}^{K}\int_0^1 c^k_{ij\chi}(l^k;\phi)\left[\mu^k_{xij}x^k_{cj}(l^k;\phi)+(1-\mu^k_{xij})x^k_{nj}(l^k;\phi)\right]dl^k$$
$$=\sum_{k=1}^{K}\frac{\rho^k}{1-\rho^k}(1-\alpha^k)\sum_{i=1}^{J}\sum_{\chi\in\{V,O\}}\left(\mu^k_{xij}+(1-\mu^k_{xij})(1-\beta^k_{ij\chi})\left(\frac{\zeta^k_{ij}}{\zeta^k_{ij\chi}}\right)\right)\left(\Xi^k_{ij\chi}\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}\bar{z}^k_{ij\chi}.$$

Next, using the expressions for $\bar{z}^k_{ij\chi}$ from (A.21), $\Xi^k_{ij\chi}$ from (15), and $X^k_j(\phi)$ from (21), one can show that:

$$\frac{\rho^k}{1-\rho^k}\left(\Xi^k_{ij\chi}\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}\bar{z}^k_{ij\chi} \;=\; (1-\alpha)\rho\eta^k R_j(\phi)\pi^k_{ij}\pi^k_{\chi|ij}(\Upsilon^k_j)^{-1}. \tag{A.23}$$

We plug this into the expressions just derived for factor payments for headquarter and supplier tasks respectively. Gathering terms, total factor payments – summed across headquarter and suppliers tasks in all input industries – are then equal to:

$$(1-\alpha)\,R_j(\phi)\sum_{k=1}^{K}\frac{\rho\eta^k}{\Upsilon^k_j}\sum_{i=1}^{J}\sum_{\chi\in\{V,O\}}\pi^k_{ij}\pi^k_{\chi|ij}\left(\alpha^k\mu^k_{hij}+(1-\alpha^k)\mu^k_{xij}+\left(\frac{\zeta^k_{ij}}{\zeta^k_{ij\chi}}\right)\left[\alpha^k(1-\mu^k_{hij})\beta^k_{ij\chi}+(1-\alpha^k)(1-\mu^k_{xij})(1-\beta^k_{ij\chi})\right]\right)$$
$$=(1-\alpha)\,R_j(\phi)\sum_{k=1}^{K}\frac{\rho\eta^k}{\Upsilon^k_j}\sum_{i=1}^{J}\sum_{\chi\in\{V,O\}}\pi^k_{ij}\pi^k_{\chi|ij}\left(\frac{\zeta^k_{ij}-(1-\rho^k)}{\rho^k}+\left(\frac{\zeta^k_{ij}}{\zeta^k_{ij\chi}}\right)\frac{1-\zeta^k_{ij\chi}}{\rho^k}\right)$$
$$=(1-\alpha)\,R_j(\phi)\sum_{k=1}^{K}\frac{\rho\eta^k}{\rho^k}\frac{1}{\Upsilon^k_j}\sum_{i=1}^{J}\sum_{\chi\in\{V,O\}}\pi^k_{ij}\pi^k_{\chi|ij}\frac{\zeta^k_{ij}}{\zeta^k_{ij\chi}}-(1-\rho^k)\pi^k_{ij}\pi^k_{\chi|ij}$$
$$=(1-\alpha)\,R_j(\phi)\sum_{k=1}^{K}\frac{\rho\eta^k}{\rho^k}\left(1-(1-\rho^k)(\Upsilon^k_j)^{-1}\right).$$

Note that in the third-to-last step above, we have made use of the definitions of $\zeta^k_{ij\chi}$ and ζ^k_{ij} respectively from equations (13) and (14). In the second-to-last step, we have then applied the definition of Υ^k_j from (23), as well as the fact that the sum of the sourcing shares $\sum_{i=1}^{J}\sum_{\chi=V,O}\pi^k_{ij}\pi^k_{\chi|ij}$ is equal to 1.

The firm's payoff function can therefore be expressed as: $F_j(\phi)=\varpi_j R_j(\phi)-w_jL_j(\phi)$, where recall that: $\varpi_j=1-(1-\alpha)\sum_{k=1}^{K}\frac{\rho\eta^k}{\rho^k}\left(1-(1-\rho^k)(\Upsilon^k_j)^{-1}\right)$, as stated in equation (24) in the main text. Moreover: $\zeta^k_{ij}-(1-\rho^k)\zeta^k_{ij\chi}=(1-\rho^k)(1-\zeta^k_{ij\chi})+\rho^k\alpha^k\mu^k_{hij}+\rho^k(1-\alpha^k)\mu^k_{xij}\ \geq\ 0$, so that $\zeta^k_{ij}/\zeta^k_{ij\chi}\geq(1-\rho^k)$. We therefore have: $\Upsilon^k_j=\sum_{i=1}^{J}\sum_{\chi\in\{V,O\}}\pi^k_{ij}\pi^k_{\chi|ij}(\zeta^k_{ij}/\zeta^k_{ij\chi})\geq 1-\rho^k$, and hence that: $1-(1-\rho^k)(\Upsilon^k_j)^{-1}\in[0,1)$. It follows that: $(1-\alpha)\sum_{k=1}^{K}\frac{\rho\eta^k}{\rho^k}\left(1-(1-\rho^k)(\Upsilon^k_j)^{-1}\right)<(1-\alpha)\sum_{k=1}^{K}\frac{\rho\eta^k}{\rho^k}<1$, where recall that the last inequality is a parameter restriction from footnote 10 in the main paper that was made to ensure that total factor payments do not exhaust the firm's revenue, $R_j(\phi)$. Thus, ϖ_j lies in the interval $(0,1]$.

Assembly labor: Recall that $R_j(\phi)=A_j^{1-\rho}q_j(\phi)^\rho$, where $q_j(\phi)=\phi L_j(\phi)^\alpha(\prod_{k=1}^{K}\left(X^k_j(\phi)\right)^{\eta^k})^{1-\alpha}$ from the production function spelled out in equation (2). Plugging in the expression for the industry-k composite input

$X_j^k(\phi)$ from (21), and simplifying, we have:

$$R_j(\phi)^{1-\rho(1-\alpha)} = A_j^{1-\rho}\phi^\rho L_j(\phi)^{\alpha\rho}\prod_{k=1}^{K}\left((1-\alpha)\rho\eta^k(\alpha^k/s_j)^{\alpha^k}(1-\alpha^k)^{1-\alpha^k}(\bar{\Gamma}^k)^{\frac{1-\rho^k}{\rho^k}}(\Phi_j^k)^{\frac{1-\alpha^k}{\theta^k}}(\Upsilon_j^k)^{\frac{1-\rho^k}{\rho^k}}\right)^{\rho\eta^k(1-\alpha)}.$$

Log differentiating this, it follows that $\frac{dR_j(\phi)}{dL_j(\phi)} = \frac{\alpha\rho}{1-\rho(1-\alpha)}\frac{R_j(\phi)}{L_j(\phi)}$. The final-good firm chooses the amount of assembly labor $L_j(\phi)$ to maximize $F_j(\phi) = \varpi_j R_j(\phi) - w_j L_j(\phi)$. Taking the first-order condition with respect to $L_j(\phi)$ and applying this expression for $\frac{dR_j(\phi)}{dL_j(\phi)}$, one immediately obtains equation (25):

$$L_j(\phi) = \frac{\alpha\rho}{1-\rho(1-\alpha)}\frac{\varpi_j}{w_j}R_j(\phi).$$

Output: Substituting for $X_j^k(\phi)$ from (21) and $L_j(\phi)$ from (25) into the production function (2), we have:

$$q_j(\phi) = \phi\rho(1-\alpha)^{1-\alpha}\left(\frac{\alpha}{1-\rho(1-\alpha)}\frac{\varpi_j}{w_j}\right)^\alpha R_j(\phi)\prod_{k=1}^{K}\left[(\alpha^k/s_j)^{\alpha^k}(1-\alpha^k)^{1-\alpha^k}\eta^k(\bar{\Gamma}^k)^{\frac{1-\rho^k}{\rho^k}}(\Phi_j^k)^{\frac{1-\alpha^k}{\theta^k}}(\Upsilon_j^k)^{\frac{1-\rho^k}{\rho^k}}\right]^{\eta^k(1-\alpha)}.$$

Replacing $R_j(\phi) = A_j^{1-\rho}q_j(\phi)^\rho$ on the right-hand side of the above, we can re-write $q_j(\phi)$ as:

$$q_j(\phi) = A_j(\phi\rho)^{\frac{1}{1-\rho}}(1-\alpha)^{\frac{1-\alpha}{1-\rho}}\left(\frac{\alpha}{1-\rho(1-\alpha)}\frac{\varpi_j}{w_j}\right)^{\frac{\alpha}{1-\rho}}\prod_{k=1}^{K}\left[(\alpha^k/s_j)^{\alpha^k}(1-\alpha^k)^{1-\alpha^k}\eta^k(\bar{\Gamma}^k)^{\frac{1-\rho^k}{\rho^k}}(\Phi_j^k)^{\frac{1-\alpha^k}{\theta^k}}(\Upsilon_j^k)^{\frac{1-\rho^k}{\rho^k}}\right]^{\eta^k\frac{1-\alpha}{1-\rho}},$$

which is the expression for output in equation (26) in the main paper.

Welfare: From equation (1), welfare is a CES aggregate over final-good varieties consumed. Since all goods assembled in country-j are consumed domestically, welfare is equal to:

$$U_j = \left(N_j\int_\phi q_j(\phi)^\rho dG_j(\phi)\right)^{\frac{1}{\rho}}, \tag{A.24}$$

where N_j is the measure of final-good producers in country j, held fixed in the baseline model. We therefore plug in $q_j(\phi)$ from equation (26) into (A.24) in order to evaluate welfare. Since $q_j(\phi)$ in (26) is a function of aggregate demand A_j, the resulting welfare expression after this substitution also depends on A_j.

Recall though from Section 2 that $A_j = E_j P_j^{\frac{\rho}{1-\rho}}$, where E_j is country-j expenditure on final goods, and P_j is the ideal price index with: $P_j^{\frac{\rho}{1-\rho}} = (N_j\int_\phi p_j(\phi)^{-\frac{\rho}{1-\rho}}dG_j(\phi))^{-1} = (N_j\int_\phi(q_j(\phi)/A_j)^\rho dG_j(\phi))^{-1}$, where we have applied here the fact that $q_j(\phi) = A_j p_j(\phi)^{-\frac{1}{1-\rho}}$. That $A_j = E_j P_j^{\frac{\rho}{1-\rho}}$ then implies: $A_j = (E_j)^{\frac{1}{1-\rho}}(N_j\int_\phi q_j(\phi)^\rho dG_j(\phi))^{-\frac{1}{1-\rho}}$. Substituting into this the expression for $q_j(\phi)$ from equation (26) and collecting terms in A_j on the left-hand side, we obtain:

$$A_j = \frac{E_j}{N_j}(\rho\bar{\phi}_j)^{-\frac{\rho}{1-\rho}}(1-\alpha)^{-\frac{\rho(1-\alpha)}{1-\rho}}\left(\frac{\alpha}{1-\rho(1-\alpha)}\frac{\varpi_j}{w_j}\right)^{-\frac{\rho\alpha}{1-\rho}}\prod_{k=1}^{K}\left[(\alpha^k/s_j)^{\alpha^k}(1-\alpha^k)^{1-\alpha^k}\eta^k(\bar{\Gamma}^k)^{\frac{1-\rho^k}{\rho^k}}(\Phi_j^k)^{\frac{1-\alpha^k}{\theta^k}}(\Upsilon_j^k)^{\frac{1-\rho^k}{\rho^k}}\right]^{-\frac{\rho(1-\alpha)}{1-\rho}\eta^k}, \tag{A.25}$$

where: $\bar{\phi}_j \equiv (\int_\phi \phi^{\frac{\rho}{1-\rho}}dG_j(\phi))^{\frac{1-\rho}{\rho}}$ is a CES aggregate of firm core productivity levels, evaluated over the underlying distribution $G_j(\phi)$ of these productivities for country j.

We take this last expression for A_j and substitute it into equation (26) for $q_j(\phi)$, and in turn plug the resulting expression for $q_j(\phi)$ into (A.24). After simplifying, this yields:

$$U_j = (N_j)^{\frac{1-\rho}{\rho}}\rho E_j\bar{\phi}_j(1-\alpha)^{1-\alpha}\left(\frac{\alpha}{1-\rho(1-\alpha)}\frac{\varpi_j}{w_j}\right)^\alpha\prod_{k=1}^{K}\left[(\alpha^k/s_j)^{\alpha^k}(1-\alpha^k)^{1-\alpha^k}\eta^k(\bar{\Gamma}^k)^{\frac{1-\rho^k}{\rho^k}}(\Phi_j^k)^{\frac{1-\alpha^k}{\theta^k}}(\Upsilon_j^k)^{\frac{1-\rho^k}{\rho^k}}\right]^{\eta^k(1-\alpha)}.$$

As a final step, we make use of the fact that from setting $i = j$ in (19), $\Phi_j^k = T_j^k(w_j)^{-\theta^k}(B_{jj}^k)^{\theta^k}/\pi_{jj}^k$. Replacing Φ_j^k in the above and simplifying delivers the welfare expression in equation (27) in the main text.

Trade flows: Recall that we adopt the position that trade flows as observed in the data reflect the value of the items that are shipped at factor costs. With this in mind, using the expressions for $x_{cj}^k(l^k;\phi)$ and $x_{nj}^k(l^k;\phi)$ from (12), we compute firm-level trade flows from country i under organizational mode χ as:

$$\begin{aligned} t_{ij\chi}^k(\phi) &= \int_{l^k\in\Omega_{ij\chi}^k} c_{ij\chi}^k \left(\mu_{xij}^k x_{cj}^k(l^k;\phi) + (1-\mu_{xij}^k) x_{nj}^k(l^k;\phi)\right) \, dl^k \\ &= \int_{l^k\in\Omega_{ij\chi}^k} (1-\alpha^k) \left(\mu_{xij}^k + (1-\mu_{xij}^k)(1-\beta_{ij\chi}^k)\frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k}\right) \frac{\rho^k}{1-\rho^k} \left(\Xi_{ij\chi}^k Z_{ij\chi}^k(l^k;\phi)\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} \, dl^k \\ &= (1-\alpha^k) \left(\mu_{xij}^k + (1-\mu_{xij}^k)(1-\beta_{ij\chi}^k)\frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k}\right) \frac{\rho^k}{1-\rho^k} (\Xi_{ij\chi}^k)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} \times \mathbb{E}_{l^k}\left[\tilde{Z}_{ij\chi}^k(l^k;\phi)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}\right], \end{aligned}$$

where $\Omega_{ij\chi}^k$ is the subset of input varieties on the unit interval, and $\tilde{Z}_{ij\chi}^k(l^k;\phi)$ denotes the supplier draw, when (i,χ) is the optimal sourcing mode.

We plug in the expression for $(\Xi_{ij\chi}^k)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} \bar{z}_{ij\chi}^k$ from equation (A.23) in the above to get:

$$t_{ij\chi}^k(\phi) = (1-\alpha)\rho\eta^k R_j(\phi)(1-\alpha^k) \left(\mu_{xij}^k + (1-\mu_{xij}^k)(1-\beta_{ij\chi}^k)\frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k}\right) (\Upsilon_j^k)^{-1} \pi_{ij}^k \pi_{\chi|ij}^k.$$

Aggregating over all country-j firms then yields:

$$t_{ij\chi}^k = (1-\alpha)\rho\eta^k(1-\alpha^k)(\Upsilon_j^k)^{-1} \left(\mu_{xij}^k + (1-\mu_{xij}^k)(1-\beta_{ij\chi}^k)\frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k}\right) \pi_{ij}^k \pi_{\chi|ij}^k N_j \int_\phi R_j(\phi) dG_j(\phi).$$

Bearing in mind that: $E_j = N_j \int_\phi R_j(\phi) dG_j(\phi)$, and plugging in for π_{ij}^k and $\pi_{\chi|ij}^k$ from (19), we obtain the expression for bilateral industry-level trade flows by organizational mode reported as (30) in the main paper:

$$t_{ij\chi}^k = (1-\alpha)\rho\eta^k(1-\alpha^k)\frac{E_j}{\Upsilon_j^k \Phi_j^k} \times T_i^k(w_i)^{-\theta^k} \times (B_{ij}^k)^{-\frac{\theta^k\lambda_i}{1-\lambda_i}} (d_{ij}^k)^{-\theta^k} \times \left(\mu_{xij}^k + (1-\mu_{xij}^k)(1-\beta_{ij\chi}^k)\frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k}\right) (B_{ij\chi}^k)^{\frac{\theta^k}{1-\lambda_i}}.$$

Factor market-clearing: We start with the factor market for labor, that is used either in supplier tasks or assembly. First, using $L_j(\phi)$ from (25), the amount of assembly labor employed in country j is given by:

$$N_j \int_\phi L_j(\phi) \, dG_j(\phi) = \frac{\alpha\rho}{w_j} \frac{\varpi_j}{1-\rho(1-\alpha)} N_j \int_\phi R_j(\phi) \, dG_j(\phi) = \frac{\alpha\rho}{w_j} \frac{\varpi_j}{1-\rho(1-\alpha)} E_j.$$

Second, country-j input suppliers also employ labor to produce input varieties that are then sourced by firms around the world (including by country-j's own final-good producers). The amount of labor employed for this latter purpose in country j is obtained by summing over the labor used to provide inputs to final-good producers in country $m = \{1,\ldots,J\}$ under sourcing mode (j,χ), where $\chi \in \{V,O\}$. Let $\Omega_{jm\chi}^k$ be the set of input varieties $l^k \in [0,1]$ from industry k for which sourcing mode (j,χ) is optimal for a final-good firm headquartered in country m. Moreover, for a country-m firm with core productivity ϕ, let $\tilde{Z}_{jm\chi}^k(l^k;\phi)$ denote the supplier draw when (j,χ) is the optimal sourcing mode for input variety l^k. The units of labor demanded by input suppliers in country j is then given by:

$$\begin{aligned} &\sum_{k=1}^K \sum_{m=1}^J \sum_{\chi\in\{V,O\}} N_m \int_\phi \int_{l^k\in\Omega_{jm\chi}^k} \frac{d_{jm}^k(\mu_{xjm}^k x_{cm}^k(l^k;\phi) + (1-\mu_{xjm}^k) x_{nm}^k(l^k;\phi))}{\tilde{Z}_{jm\chi}^k(l^k;\phi)} \, dl^k dG_m(\phi) \\ &= \sum_{k=1}^K \sum_{m=1}^J \sum_{\chi\in\{V,O\}} N_m \int_\phi \int_{l^k\in\Omega_{jm\chi}^k} \frac{1-\alpha^k}{w_j} \left(\mu_{xjm}^k + (1-\mu_{xjm}^k)(1-\beta_{jm\chi}^k)\frac{\zeta_{jm}^k}{\zeta_{jm\chi}^k}\right) \frac{\rho^k}{1-\rho^k} (\Xi_{jm\chi}^k \tilde{Z}_{jm\chi}^k(l^k;\phi))^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} dl^k dG_m(\phi) \\ &= \sum_{k=1}^K \sum_{m=1}^J \sum_{\chi\in\{V,O\}} N_m \int_\phi \frac{1-\alpha^k}{w_j} \left(\mu_{xjm}^k + (1-\mu_{xjm}^k)(1-\beta_{jm\chi}^k)\frac{\zeta_{jm}^k}{\zeta_{jm\chi}^k}\right) \frac{\rho^k}{1-\rho^k} (\Xi_{jm\chi}^k)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} \mathbb{E}_{l^k}\left[\tilde{Z}_{jm\chi}^k(l^k;\phi)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}\right] dG_m(\phi), \end{aligned}$$

where we have made use of the expressions for the optimal task investment levels in (12). Substituting in the expression for $(\Xi^k_{jm\chi})^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}\bar{z}^k_{jm\chi}$ implied by (A.23), and making use of the fact that: $E_m = N_m \int_\phi R_m(\phi)dG_m(\phi)$, the above simplifies to:

$$\sum_{k=1}^{K}\sum_{m=1}^{J}\sum_{\chi\in\{V,O\}}(1-\alpha)\rho\eta^k\frac{E_m}{\Upsilon^k_m}\pi^k_{jm}\pi^k_{\chi|jm}\frac{1-\alpha^k}{w_j}\left(\mu^k_{xjm}+(1-\mu^k_{xjm})(1-\beta^k_{jm\chi})\frac{\zeta^k_{jm}}{\zeta^k_{jm\chi}}\right).$$

We sum up the above two components of labor demand in country j – across its use in assembly and in supplier tasks – and equate this to the endowment of labor, $\bar{L}_j$; this gives us the labor market-clearing condition spelled out in (32) in the main paper.

We turn next to the factor market for capital in country j. This factor is used only to provide headquarter services for final-good firms based in country j, that are combined with supplier services in order to yield the input varieties $l^k \in [0,1]$. We therefore sum over the capital used across input varieties, taking into account the respective shares of $l^k \in [0,1]$ that will be optimally sourced under each of the $2J$ possible sourcing modes (i,χ). A similar set of algebraic steps can be applied as in the above derivation performed for the labor market-clearing condition, to obtain an expression for the total demand for capital in country j:

$$\begin{aligned}&\sum_{k=1}^{K}\sum_{i=1}^{J}\sum_{\chi\in\{V,O\}}N_j\int_\phi\int_{l^k=0}^{1}\left[\mu^k_{hij}h^k_{cj}(l^k;\phi)+(1-\mu^k_{hij})h^k_{nj}(l^k;\phi)\right]dl^k dG_j(\phi)\\ =&\sum_{k=1}^{K}\sum_{i=1}^{J}\sum_{\chi\in\{V,O\}}(1-\alpha)\rho\eta^k\frac{E_j}{\Upsilon^k_j}\pi^k_{ij}\pi^k_{\chi|ij}\frac{\alpha^k}{s_j}\left(\mu^k_{hij}+(1-\mu^k_{hij})\beta^k_{ij\chi}\frac{\zeta^k_{ij}}{\zeta^k_{ij\chi}}\right).\end{aligned}$$

Note that we have used the optimal headquarter task investment levels in (12), as well as the expression for $(\Xi^k_{ij\chi})^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}\bar{z}^k_{ij\chi}$ from (A.23), to derive the above. Equating this to the endowment $\bar{K}_j$ yields the capital market-clearing condition in (33).

Aggregate expenditure: Aggregate expenditure in country j, E_j, is the sum of aggregate income, denoted by I_j, and the current account deficit, D_j, which we will treat as exogenously fixed as in Dekle et al. (2008). Note that a positive deficit, $D_j > 0$, means that a country's exports are less than its imports.

Aggregate income, I_j, is in turn pinned down as follows. There are two primary factors of production in this economy: (i) capital, which is used in headquarter tasks; as well as (ii) labor, which is used in supplier tasks and in final-good assembly. Let $\bar{K}_j$ and $\bar{L}_j$ denote respectively the country-j endowments of these two factors. We further assume that firm profits are entirely rebated to country-j households via a domestic asset market and that there is no cross-border trade in assets. Therefore, aggregate income in country j is:

$$\begin{aligned}I_j &= w_j\bar{L}_j + s_j\bar{K}_j + N_j\left(\int_\phi \varpi_j R_j(\phi)dG_j(\phi) - \int_\phi w_j L_j(\phi)dG_j(\phi)\right)\\ &= w_j\bar{L}_j + s_j\bar{K}_j + \left(1-\frac{\alpha\rho}{1-\rho(1-\alpha)}\varpi_j\right)N_j\int_\phi R_j(\phi)dG_j(\phi),\end{aligned}$$

where we have made use of the expression for $L_j(\phi)$ from (25).

Market-clearing implies that aggregate expenditure is equal to firm revenues summed across all final-good producers in the country j: $E_j = N_j\int_\phi R_j(\phi)dG_j(\phi)$. Using this in the above expression for I_j, together with the fact that $E_j = I_j + D_j$, it is straightforward to show that E_j can be written in terms of factor payments and the current account deficit as reported in equation (34) in the main paper.

A.3 Equilibrium System

We spell out the equilibrium system of equations in our J country, K industry world with trade under contracting frictions:

$$\zeta^k_{ij} = 1-\rho^k+\rho^k\alpha^k\mu^k_{hij}+\rho^k(1-\alpha^k)\mu^k_{xij} \tag{A.26}$$

$$\zeta^k_{ij\chi} = 1 - \rho^k \alpha^k \left(1 - \mu^k_{hij}\right) \beta^k_{ij\chi} - \rho^k (1-\alpha^k) \left(1 - \mu^k_{xij}\right) \left(1 - \beta^k_{ij\chi}\right) \tag{A.27}$$

$$B^k_{ij\chi} = \left(\frac{\zeta^k_{ij\chi}}{\zeta^k_{ij}}\right)^{\frac{\zeta^k_{ij}}{\rho^k(1-\alpha^k)}} \left(\beta^k_{ij\chi}\right)^{\frac{\alpha^k}{1-\alpha^k}\left(1-\mu^k_{hij}\right)} \left(1 - \beta^k_{ij\chi}\right)^{1-\mu^k_{xij}} \tag{A.28}$$

$$B^k_{ij} = \left(\left(B^k_{ijV}\right)^{\frac{\theta^k}{1-\lambda_i}} + \left(B^k_{ijO}\right)^{\frac{\theta^k}{1-\lambda_i}}\right)^{\frac{1-\lambda_i}{\theta^k}} \tag{A.29}$$

$$\pi^k_{\chi|ij} = \frac{\left(B^k_{ij\chi}\right)^{\frac{\theta^k}{1-\lambda_i}}}{\left(B^k_{ijV}\right)^{\frac{\theta^k}{1-\lambda_i}} + \left(B^k_{ijO}\right)^{\frac{\theta^k}{1-\lambda_i}}} \tag{A.30}$$

$$\pi^k_{ij} = T^k_i (d^k_{ij} w_i)^{-\theta^k} (B^k_{ij})^{\theta^k} / \Phi^k_j \tag{A.31}$$

$$\Phi^k_j = \sum_{i=1}^{J} T^k_i (d^k_{ij} w_i)^{-\theta^k} (B^k_{ij})^{\theta^k} \tag{A.32}$$

$$\Upsilon^k_j = \sum_{i=1}^{J} \sum_{\chi=\{V,O\}} \frac{\zeta^k_{ij}}{\zeta^k_{ij\chi}} \pi^k_{ij} \pi^k_{\chi|ij} \tag{A.33}$$

$$\varpi_j = 1 - (1-\alpha) \sum_{k=1}^{K} \frac{\rho \eta^k}{\rho^k} \left(1 - (1-\rho^k)/\Upsilon^k_j\right) \tag{A.34}$$

$$E_j = \frac{w_j \bar{L}_j + s_j \bar{K}_j + D_j}{1 - \frac{1-\rho}{1-\rho(1-\alpha)} \varpi_j} \tag{A.35}$$

$$w_j \bar{L}_j = \frac{\alpha \rho \varpi_j}{1 - \rho(1-\alpha)} E_j + (1-\alpha)\rho \sum_{k=1}^{K} \sum_{m=1}^{J} \sum_{\chi \in \{V,O\}} \eta^k (1-\alpha^k) \frac{E_m}{\Upsilon^k_m} \pi^k_{jm} \pi^k_{\chi|jm} \left(\mu^k_{xjm} + (1-\mu^k_{xjm})(1-\beta^k_{jm\chi}) \frac{\zeta^k_{jm}}{\zeta^k_{jm\chi}}\right) \tag{A.36}$$

$$s_j \bar{K}_j = (1-\alpha)\rho \sum_{k=1}^{K} \sum_{i=1}^{J} \sum_{\chi \in \{V,O\}} \eta^k \alpha^k \frac{E_j}{\Upsilon^k_j} \pi^k_{ij} \pi^k_{\chi|ij} \left(\mu^k_{hij} + (1-\mu^k_{hij}) \beta^k_{ij\chi} \frac{\zeta^k_{ij}}{\zeta^k_{ij\chi}}\right) \tag{A.37}$$

As $i, j \in \{1, \ldots, J\}$ and $k \in \{1, \ldots, K\}$, equations (A.26), (A.29), and (A.31)-(A.37) comprise $3J^2K + 2JK + 4J$ equations. With $\chi \in \{V, O\}$, equations (A.27) and (A.28) are another $2 \times 2J^2K = 4J^2K$ equations. Note that equation (A.30) comprises just J^2K equations, rather than $2J^2K$, since $\pi^k_{V|ij} + \pi^k_{O|ij} = 1$ for any given i, j, and k. The full equilibrium system thus contains $8J^2K + 2JK + 4J$ equations.

The variables to be pinned down are: $\zeta^k_{ij\chi}$, $B^k_{ij\chi}$ ($2J^2K$ each); ζ^k_{ij}, B^k_{ij}, $\pi^k_{V|ij}$, π^k_{ij} (J^2K each); Φ^k_j, Υ^k_j (JK each); as well as ϖ_j, E_j, w_j, s_j (J each). There are thus as many equations as unknowns; with a choice of numeraire, one equation can be dropped (based on Walras' Law).

B Estimation

B.1 Transition to Empirics

Pseudo-Maximum Likelihood (PML): We derive the moment condition for the intrafirm trade share (by value) via a Poisson PML approach. Recall from the empirical model in Section (4.1) that the value of observed trade flows by organizational mode is given by: $\tilde{t}^k_{ij\chi} = a^k_{ij} a^k_{ij\chi} \epsilon^k_{ij\chi}$, with $\mathbb{E}[\tilde{t}^k_{ij\chi} | a^k_{ij\chi}, a^k_{ij}] = a^k_{ij} a^k_{ij\chi}$. The associated log Poisson pseudo-maximum likelihood function is therefore equal (up to an additive constant) to:

$$\sum_{i,j,k} \sum_{\chi \in \{V,O\}} -a^k_{ij} a^k_{ij\chi} + \tilde{t}^k_{ij\chi} \log(a^k_{ij} a^k_{ij\chi}).$$

The first-order condition of the above with respect to a^k_{ij} implies that the Poisson PML estimator, $\hat{a}^k_{ij}$, for this country-pair-by-industry fixed effect satisfies:

$$\hat{a}^k_{ij} \sum_{\chi \in \{V,O\}} a^k_{ij\chi} = \sum_{\chi \in \{V,O\}} \tilde{t}^k_{ij\chi}, \text{ so that: } \hat{a}^k_{ij} = \frac{\sum_{\chi \in \{V,O\}} \tilde{t}^k_{ij\chi}}{\sum_{\chi \in \{V,O\}} a^k_{ij\chi}}.$$

The concavity of the log function ensures that the solution $\hat{a}^k_{ij}$ is the global maxima. We now substitute $\hat{a}^k_{ij}$ back into $\tilde{t}^k_{ij\chi} = a^k_{ij} a^k_{ij\chi} \epsilon^k_{ij\chi}$ for a^k_{ij}; a quick rearrangement then implies:

$$\frac{\tilde{t}^k_{ijV}}{\sum_{\chi \in \{V,O\}} \tilde{t}^k_{ij\chi}} = \frac{a^k_{ijV}}{\sum_{\chi \in \{V,O\}} a^k_{ij\chi}} \epsilon^k_{ijV}. \tag{B.1}$$

Since $\mathbb{E}[\epsilon^k_{ij\chi} | a^k_{ij\chi}, a^k_{ij}] = 1$, this delivers the moment condition: $\mathbb{E}[\tilde{t}^k_{ijV} / (\tilde{t}^k_{ijV} + \tilde{t}^k_{ijO}) | a_{ijV}, a_{ijO}, a^k_{ij}] = a^k_{ijV} / (a^k_{ijV} + a^k_{ijO})$. Note that this derivation rests on the fact that the Poisson PML estimator satisfies the following "adding-up" property (c.f., Arvis and Shepherd, 2013; Fally, 2015): $\hat{a}^k_{ij} a^k_{ijV} + \hat{a}^k_{ij} a^k_{ijO} = \tilde{t}^k_{ijV} + \tilde{t}^k_{ijO}$. In other words, predicted trade flows are equal to observed trade flows when summed across organizational modes for a given country-pair-by-industry.

Alternative approach: We provide an alternative derivation of the moment condition, under the premise that the trade flows by organizational mode observed in the data, $\tilde{t}^k_{ij\chi}$, are realizations of a Poisson distribution with mean: $\mathbb{E}[\tilde{t}^k_{ij\chi} | a^k_{ij\chi}, a^k_{ij}] = a^k_{ij} a^k_{ij\chi}$. We invoke the following property: If X_1 and X_2 are two independent Poisson random variables with mean λ_1 and λ_2 respectively, then the distribution of X_1 conditional on the sum $X_1 + X_2$ is binomial with the number of trials being the realized value of $X_1 + X_2$ and the success probability being $\lambda_1 / (\lambda_1 + \lambda_2)$; see, for example, Rohatgi and Saleh (2000). In our setting, this implies that conditional on $\tilde{t}^k_{ijV} + \tilde{t}^k_{ijO} = \tilde{t}^k_{ij}$, the distribution of $\tilde{t}^k_{ijV}$ is binomial with $\tilde{t}^k_{ij}$ being the number of the trials and $a^k_{ij} a^k_{ijV} / (a^k_{ij} a^k_{ijV} + a^k_{ij} a^k_{ijO}) = a^k_{ijV} / (a^k_{ijV} + a^k_{ijO})$ being the success probability. A binomial distribution with $\tilde{t}^k_{ij}$ trials is equivalent to the sum of $\tilde{t}^k_{ij}$ independent Bernoulli distributions with the same success probability. Therefore, $\tilde{t}^k_{ijV} / \tilde{t}^k_{ij}$ conditional on $\tilde{t}^k_{ij}$ follows a Bernoulli distribution with success probability – and hence expected value – equal to $a^k_{ijV} / (a^k_{ijV} + a^k_{ijO})$. This yields the moment condition: $\mathbb{E}[\tilde{t}^k_{ijV} / \tilde{t}^k_{ij} | \tilde{t}^k_{ij}] = a^k_{ijV} / (a^k_{ijV} + a^k_{ijO})$.

B.2 Properties of the Intrafirm Trade Share by Value

From the trade flow expression for $t^k_{ij\chi}$ in (30), the intrafirm trade share – by value – for country j's sourcing from country i in industry k is given by:

$$\frac{t^k_{ijV}}{t^k_{ijV} + t^k_{ijO}} = \frac{(b^k_{ijV} / b^k_{ijO})(B^k_{ijV} / B^k_{ijO})^{\frac{\theta^k}{1-\lambda_i}}}{(b^k_{ijV} / b^k_{ijO})(B^k_{ijV} / B^k_{ijO})^{\frac{\theta^k}{1-\lambda_i}} + 1}, \tag{B.2}$$

where $b^k_{ij\chi} \equiv \mu^k_{xij} + (1 - \mu^k_{xij})(1 - \beta^k_{ij\chi}) \frac{\zeta^k_{ij}}{\zeta^k_{ij\chi}}$, for $\chi \in \{V, O\}$.

Based on (20), we can see that $(B^k_{ijV}/B^k_{ijO})^{\frac{\theta^k}{1-\lambda_i}}$ governs the probability of sourcing through an intrafirm transaction rather than at arm's length (i.e., $\pi^k_{V|ij}$); in other words, we can view $(B^k_{ijV}/B^k_{ijO})^{\frac{\theta^k}{1-\lambda_i}}$ as a term that governs the extensive margin of intrafirm sourcing for a given input variety. b^k_{ijV}/b^k_{ijO} then acts to convert $\pi^k_{V|ij}$ into the intrafirm trade share by value; we can thus interpret b^k_{ijV}/b^k_{ijO} as an adjustment term that accounts for the intensive margin of intrafirm sourcing, which captures the value of supplier inputs transacted conditional on integration being chosen relative to the value transacted conditional on outsourcing.

It is useful to note that this adjustment term satisfies: $b^k_{ijV}/b^k_{ijO} \leq 1$. This follows from the fact that: $\frac{1-\beta^k_{ijV}}{\zeta^k_{ijV}} - \frac{1-\beta^k_{ijO}}{\zeta^k_{ijO}} = -\frac{\beta^k_{ijV}-\beta^k_{ijO}}{\zeta^k_{ijV}\zeta^k_{ijO}}(1-\rho^k\alpha^k(1-\mu^k_{hij})) < 0$, with equality holding in the limit case where δ^k_{ij} tends to 0 (and there is no difference in the bargaining shares under integration versus outsourcing). This property is intuitive: Bear in mind that the value of trade flows is determined by total factor costs incurred in supplier tasks. Then, all else constant, the value of supplier inputs under outsourcing is higher than that under integration, as the former organizational mode better incentivizes supplier effort.

In what follows, we derive comparative statics results for the intrafirm trade share by value, with respect to α^k, μ^k_{hij}, μ^k_{xij}, β^k_{ijO}, and δ^k_{ij}. From (B.2), these properties are pinned down by the corresponding behavior of $(b^k_{ijV}/b^k_{ijO})(B^k_{ijV}/B^k_{ijO})^{\frac{\theta^k}{1-\lambda_i}}$ with respect to each parameter. The results below parallel those presented in Lemma 2 for $\pi^k_{V|ij}$ (the intrafirm trade share by count of input varieties). As we will see, the intrafirm trade share by value inherits the comparative statics behavior of $\pi^k_{V|ij}$ over a wide range of empirically relevant parameter values. It turns out moreover that a sufficient condition – namely, that λ_i be large – is enough to ensure that the full set of comparative statics from Lemma 2 carries through analytically for the entire range of values taken on by the other model parameters. (We provide intuition for this sufficient condition below.) In our quantitative work, we indeed obtain estimates of λ_i that are close to 1, so this sufficient condition is readily satisfied in practice.

Behavior of intrafirm trade share by value with respect to α^k: We show that $t^k_{ijV}/(t^k_{ijV}+t^k_{ijO})$ from (B.2) increases with respect to α^k; the sufficient condition – that λ_i be large – is needed here to ensure that this comparative static holds over the full range of values of the other model parameters. (The derivations in the remainder of this appendix section will assume that $\mu^k_{hij}, \mu^k_{xij} \in (0,1)$, although one can readily see that the insights on comparative statics are preserved when μ^k_{hij} and μ^k_{xij} take on their limiting values of either 0 or 1.)

In the proof of Lemma 2(i) earlier in Appendix A.1, we evaluated the log derivative of B^k_{ijV}/B^k_{ijO} and showed that $(B^k_{ijV}/B^k_{ijO})^{\frac{\theta^k}{1-\lambda_i}}$ is unambiguously increasing in α^k for all $\alpha^k \in (0,1)$. We therefore focus on characterizing the behavior of b^k_{ijV}/b^k_{ijO} with respect to α^k. We have:

$$\frac{d}{d\alpha^k}\log\frac{b^k_{ijV}}{b^k_{ijO}} = \frac{(1-\mu^k_{xij})(1-\beta^k_{ijV})\frac{d}{d\alpha^k}\frac{\zeta^k_{ij}}{\zeta^k_{ijV}}}{b^k_{ijV}} - \frac{(1-\mu^k_{xij})(1-\beta^k_{ijO})\frac{d}{d\alpha^k}\frac{\zeta^k_{ij}}{\zeta^k_{ijO}}}{b^k_{ijO}},$$

where: $\frac{d}{d\alpha^k}\frac{\zeta^k_{ij}}{\zeta^k_{ij\chi}} = \frac{\zeta^k_{ij}}{\zeta^k_{ij\chi}}\rho^k[(1-\mu^k_{xij})(\frac{1}{\zeta^k_{ij}} - \frac{1-\beta^k_{ij\chi}}{\zeta^k_{ij\chi}}) + (1-\mu^k_{hij})(\frac{\beta^k_{ij\chi}}{\zeta^k_{ij\chi}} - \frac{1}{\zeta^k_{ij}})]$ for $\chi \in \{V, O\}$. When $\beta_{ijV} = \beta_{ijO}$, we clearly have $\frac{d}{d\alpha^k}\log(b^k_{ijV}/b^k_{ijO}) = 0$. Following the approach in the proof of Lemma 2(i), we proceed to study the behavior of $\frac{d}{d\alpha^k}\log(b^k_{ijV}/b^k_{ijO})$ when one increases β_{ijV} from $\beta_{ijV} = \beta_{ijO}$, with an eye on understanding under what conditions we would have $\frac{d}{d\beta^k_{ijV}}\frac{d}{d\alpha^k}(\frac{\theta^k}{1-\lambda_i}\log(B^k_{ijV}/B^k_{ijO}) + \log(b^k_{ijV}/b^k_{ijO})) > 0$, and hence also that $\frac{d}{d\alpha^k}(\frac{\theta^k}{1-\lambda_i}\log(B^k_{ijV}/B^k_{ijO}) + \log(b^k_{ijV}/b^k_{ijO})) > 0$ for $\beta_{ijV} > \beta_{ijO}$.

From the proof of Lemma 2(i), we have: $\frac{d}{d\beta^k_{ijV}}\frac{d}{d\alpha^k}\frac{\theta^k}{1-\lambda_i}\log(B^k_{ijV}/B^k_{ijO}) = \frac{\theta^k}{1-\lambda_i}\frac{1}{\rho^k(1-\alpha^k)^2}\frac{d\psi}{d\beta^k_{ijV}}$, where ψ was defined in (A.15) and an expression for $\frac{d\psi}{d\beta^k_{ijV}}$ was given in (A.16). Specifically:

$$\begin{aligned}\frac{d}{d\beta^k_{ijV}}\frac{d}{d\alpha^k}\frac{\theta^k}{1-\lambda_i}\log\frac{B^k_{ijV}}{B^k_{ijO}} = \frac{\rho^k}{(\zeta^k_{ijV})^2}\frac{\theta^k}{1-\lambda_i}\frac{1}{\rho^k(1-\alpha^k)^2}\frac{1}{\beta^k_{ijV}}\Big[&(1-\mu^k_{hij})\left(\zeta^k_{ijV}(1-\alpha^k\beta^k_{ijV}) - \zeta^k_{ij}(1-\alpha^k)\beta^k_{ijV}\right) \\ &+ (1-\mu^k_{xij})(1-\alpha^k)\beta^k_{ijV}\left(\zeta^k_{ijV}-\zeta^k_{ij}\right) \\ &+ \rho^k(1-\mu^k_{hij})(1-\mu^k_{xij})(1-\alpha^k)\left(\zeta^k_{ij}\beta^k_{ijV}-\zeta^k_{ijV}\right)\Big]. \qquad \text{(B.3)}\end{aligned}$$

Note that we have collected terms involving $(1-\mu^k_{hij})$ in the first line; terms involving $(1-\mu^k_{xij})$ in the second line; as well as terms in $\rho^k(1-\mu^k_{hij})(1-\mu^k_{xij})$ in the third line of this last equation.

We next evaluate an expression for $\frac{d}{d\beta^k_{ijV}}\frac{d}{d\alpha^k}\log(b^k_{ijV}/b^k_{ijO})$. With some algebra, one can show that:

$$\frac{d}{d\beta^k_{ijV}}\frac{d}{d\alpha^k}\log\frac{b^k_{ijV}}{b^k_{ijO}} = \frac{\mu^k_{xij}(1-\mu^k_{xij})}{(b^k_{ijV})^2}\rho^k\left[(1-\mu^k_{xij})\left(\frac{1}{\zeta^k_{ij}}-\frac{1-\beta^k_{ijV}}{\zeta^k_{ijV}}\right)+(1-\mu^k_{hij})\left(\frac{\beta^k_{ijV}}{\zeta^k_{ijV}}-\frac{1}{\zeta^k_{ij}}\right)\right]\zeta^k_{ij}\frac{d}{d\beta^k_{ijV}}\frac{1-\beta^k_{ijV}}{\zeta^k_{ijV}}$$
$$+\frac{(1-\mu^k_{xij})(1-\beta^k_{ijV})\frac{\zeta^k_{ij}}{\zeta^k_{ijV}}}{b^k_{ijV}}\rho^k\left[-(1-\mu^k_{xij})\frac{d}{d\beta^k_{ijV}}\frac{1-\beta^k_{ijV}}{\zeta^k_{ijV}}+(1-\mu^k_{hij})\frac{d}{d\beta^k_{ijV}}\frac{\beta^k_{ijV}}{\zeta^k_{ijV}}\right].$$

From straightforward differentiation, we have: $\frac{d}{d\beta^k_{ijV}}\frac{1-\beta^k_{ijV}}{\zeta^k_{ijV}} = -\frac{1-\rho^k\alpha^k(1-\mu^k_{hij})}{(\zeta^k_{ijV})^2}$, and $\frac{d}{d\beta^k_{ijV}}\frac{\beta^k_{ijV}}{\zeta^k_{ijV}} = \frac{1-\rho^k(1-\alpha^k)(1-\mu^k_{xij})}{(\zeta^k_{ijV})^2}$, which we substitute into the above expression for $\frac{d}{d\beta^k_{ijV}}\frac{d}{d\alpha^k}\log(b^k_{ijV}/b^k_{ijO})$. Define $\phi^k_{ij}\equiv\frac{\mu^k_{xij}}{b^k_{ijV}}$ to ease the notation; observe that $\phi^k_{ij}\in[0,1]$. After further simplification, we arrive at:

$$\frac{d}{d\beta^k_{ijV}}\frac{d}{d\alpha^k}\log\frac{b^k_{ijV}}{b^k_{ijO}} = \frac{\rho^k}{(\zeta^k_{ijV})^2}\Bigg[(1-\mu^k_{hij})\Big[(1-\phi^k_{ij})+\phi^k_{ij}(1-\phi^k_{ij})\frac{1-\rho^k\alpha^k(1-\mu^k_{hij})}{(1-\beta^k_{ijV})\zeta^k_{ij}}\left(\zeta^k_{ijV}-\zeta^k_{ij}\beta^k_{ijV}\right)\Big]$$
$$+(1-\mu^k_{xij})\Big[(1-\phi^k_{ij})+\phi^k_{ij}(1-\phi^k_{ij})\frac{1-\rho^k\alpha^k(1-\mu^k_{hij})}{(1-\beta^k_{ijV})\zeta^k_{ij}}\left(\zeta^k_{ij}(1-\beta^k_{ijV})-\zeta^k_{ijV}\right)\Big]$$
$$-\rho^k(1-\mu^k_{hij})(1-\mu^k_{xij})(1-\phi^k_{ij})\Bigg]. \tag{B.4}$$

This is written down in a similar format to (B.3), with the three lines corresponding to terms in $(1-\mu^k_{hij})$, $(1-\mu^k_{xij})$, and $\rho^k(1-\mu^k_{hij})(1-\mu^k_{xij})$ respectively.

Focus first on the terms in the first and third lines of (B.3). We have:

$$\begin{aligned}&(1-\mu^k_{hij})\left(\zeta^k_{ijV}(1-\alpha^k\beta^k_{ijV})-\zeta^k_{ij}(1-\alpha^k)\beta^k_{ijV}\right)+\rho^k(1-\mu^k_{hij})(1-\mu^k_{xij})(1-\alpha^k)(\zeta^k_{ij}\beta^k_{ijV}-\zeta^k_{ijV})\\ &>\rho^k(1-\mu^k_{hij})(1-\mu^k_{xij})\left[\left(\zeta^k_{ijV}(1-\alpha^k\beta^k_{ijV})-\zeta^k_{ij}(1-\alpha^k)\beta^k_{ijV}\right)+(1-\alpha^k)\left(\zeta^k_{ij}\beta^k_{ijV}-\zeta^k_{ijV}\right)\right]\\ &=\rho^k(1-\mu^k_{hij})(1-\mu^k_{xij})\alpha^k\zeta^k_{ijV}(1-\beta^k_{ijV})\\ &>0,\end{aligned}$$

where the first inequality follows from: (i) $\rho^k(1-\mu^k_{xij})\in(0,1)$, and (ii) $\zeta^k_{ijV}(1-\alpha^k\beta^k_{ijV})-\zeta^k_{ij}(1-\alpha^k)\beta^k_{ijV} > (1-\alpha^k)\beta^k_{ijV}(\zeta^k_{ijV}-\zeta^k_{ij})>0$. The sum of the terms in the first and third lines of (B.3) is thus positive.

Next, consider the terms in the first and the third lines of (B.4). Since $\zeta^k_{ijV}>\zeta^k_{ij}$ and $\beta^k_{ijV}\in(0,1)$, the term in the first line involving $(\zeta^k_{ijV}-\zeta^k_{ij}\beta^k_{ijV})$ has a positive sign. Combining the remaining term in the first line with the term in the third line, we have: $(1-\mu^k_{hij})(1-\phi^k_{ij})-\rho^k(1-\mu^k_{hij})(1-\mu^k_{xij})(1-\phi^k_{ij})>0$, since $\rho^k(1-\mu^k_{xij})\in(0,1)$. The sum of the terms in the first and third lines of (B.4) can therefore be signed unambiguously as positive.

It remains to address the second lines of (B.3) and (B.4). It turns out that there are parameter values for which the sum of these terms in $(1-\mu^k_{xij})$ takes on a negative sign, so while the sign of the first and third lines of (B.3) and (B.4) is positive, this does not guarantee that $\frac{d}{d\beta^k_{ijV}}\frac{d}{d\alpha^k}(\frac{\theta^k}{1-\lambda_i}\log(B^k_{ijV}/B^k_{ijO})+\log(b^k_{ijV}/b^k_{ijO}))$ overall will be positive: there are situations – albeit extreme ones – under which the intrafirm trade share by value can be decreasing in α^k. For example, consider: $\rho^k=0.75$, $\mu^k_{hij}=0.99$, $\mu^k_{xij}=0.01$, $\beta^k_{ijO}=0.5$, $\delta^k_{ij}=0.99$, $\theta^k=4$, and $\lambda_i=0$. The intrafirm trade share by value is equal to 1.200×10^{-9} at $\alpha^k=0$, but drops to 1.197×10^{-9} at $\alpha^k=0.05$. Notice that this is an extreme scenario with supplier inputs being highly noncontractible and the firm's bargaining share under integration being very high ($\beta^k_{ijV}=0.995$). The supplier would have very weak incentives to invest in effort, so much so that when supplier inputs are important (α^k is small), outsourcing is almost always chosen as the optimal organizational mode. A small increase in α^k would prompt some switching to integration on the extensive margin, but the lower supplier effort on the intensive margin under integration results in an overall decrease in the intrafirm trade share by value. (For slightly less extreme parameterizations, the intrafirm trade share by value readily reverts to being monotonically increasing over $\alpha^k \in [0,1]$.)

That said, one can readily derive a sufficient condition for $\frac{d}{d\beta^k_{ijV}} \frac{d}{d\alpha^k} (\frac{\theta^k}{1-\lambda_i} \log(B^k_{ijV}/B^k_{ijO}) + \log(b^k_{ijV}/b^k_{ijO}))$ to be positive. Combining the terms in the second lines of (B.3) and (B.4) yields:

$$\frac{\rho^k}{(\zeta^k_{ijV})^2}(1-\mu^k_{xij}) \left[\frac{\theta^k}{1-\lambda_i} \frac{1}{\rho^k(1-\alpha^k)} \left(\zeta^k_{ijV} - \zeta^k_{ij}\right) + (1-\phi^k_{ij}) + \phi^k_{ij}(1-\phi^k_{ij}) \frac{1-\rho^k\alpha^k(1-\mu^k_{hij})}{(1-\beta^k_{ijV})\zeta^k_{ij}} \left(\zeta^k_{ij}(1-\beta^k_{ijV}) - \zeta^k_{ijV}\right) \right].$$

Since $\zeta^k_{ijV} > \zeta^k_{ij}$, we have $\zeta^k_{ijV} - \zeta^k_{ij} > 0$, but $\zeta^k_{ij}(1-\beta^k_{ijV}) - \zeta^k_{ijV} < 0$. We therefore seek conditions under which the positive term involving $(\zeta^k_{ijV} - \zeta^k_{ij})$ in the above expression would dominate the negative term involving $(\zeta^k_{ij}(1-\beta^k_{ijV}) - \zeta^k_{ijV})$. It suffices in particular if λ_i were large: as λ_i approaches its maximum value of 1, the term $\frac{\theta^k}{1-\lambda_i} \frac{1}{\rho^k(1-\alpha^k)}(\zeta^k_{ijV} - \zeta^k_{ij})$ would tend to $+\infty$ in the limit, while the remaining terms in the above expression would remain finite and bounded. (Alternatively, it is straightforward to see that either α^k or μ^k_{xij} being large would be sufficient conditions too. When α^k approaches 1, the above expression would be positive; on the other hand, when μ^k_{xij} approaches 1, the above expression tends to zero, allowing the positive terms in the first and third lines of (B.3) and (B.4) to dominate. We nevertheless focus on λ_i being large as the key sufficient condition, since the empirical estimates of λ_i that we obtain are indeed all close to 1.)

Intuitively, if the correlation between the supplier draws under integration and outsourcing is high, this would amplify the effect of a higher α^k on the propensity toward integration on the extensive margin. Put otherwise, this dampens the scope for outsourcing to be the optimal organizational mode as it is unlikely that the supplier draw under outsourcing, z^k_{ijO}, is vastly better than that under integration, z^k_{ijV}, when there is a strong underlying positive correlation between these draws. In response to a higher α^k then, the increase in integration on the extensive margin would dominate shifts on the intensive margin.

Behavior of intrafirm trade share by value with respect to μ^k_{hij} and μ^k_{xij}: We have seen in the proof of Lemma 2(ii) that B^k_{ijV}/B^k_{ijO} is decreasing in μ^k_{hij}, while increasing in μ^k_{xij}. We show here that b^k_{ijV}/b^k_{ijO} too is decreasing in μ^k_{hij}, while increasing in μ^k_{xij}, so that the intrafirm trade share by value in (B.2) inherits these comparative static properties; moreover, these particular results hold without the need for a sufficient condition.

Consider first the log derivative of b^k_{ijV}/b^k_{ijO} with respect to μ^k_{hij}:

$$\frac{d}{d\mu^k_{hij}} \log \frac{b^k_{ijV}}{b^k_{ijO}} = \frac{(1-\mu^k_{xij})(1-\beta^k_{ijV})}{b^k_{ijV}} \frac{d}{d\mu^k_{hij}} \frac{\zeta^k_{ij}}{\zeta^k_{ijV}} - \frac{(1-\mu^k_{xij})(1-\beta^k_{ijO})}{b^k_{ijO}} \frac{d}{d\mu^k_{hij}} \frac{\zeta^k_{ij}}{\zeta^k_{ijO}}.$$

Using the quotient rule to evaluate $\frac{d}{d\mu^k_{hij}} \frac{\zeta^k_{ij}}{\zeta^k_{ij\chi}}$ for $\chi \in \{V, O\}$, and simplifying extensively, we obtain:

$$\frac{d}{d\mu^k_{hij}} \log \frac{b^k_{ijV}}{b^k_{ijO}} = \frac{\rho^k\alpha^k}{\zeta^k_{ij}} \frac{1-\mu^k_{xij}}{b^k_{ijV}b^k_{ijO}} \left[\mu^k_{xij} \left(\frac{\zeta^k_{ij}}{\zeta^k_{ijV}}(1-\beta^k_{ijV}) \left(1 - \frac{\zeta^k_{ij}}{\zeta^k_{ijV}}\beta^k_{ijV}\right) - \frac{\zeta^k_{ij}}{\zeta^k_{ijO}}(1-\beta^k_{ijO}) \left(1 - \frac{\zeta^k_{ij}}{\zeta^k_{ijO}}\beta^k_{ijO}\right) \right) \right.$$
$$\left. + (1-\mu^k_{xij})(1-\beta^k_{ijV})(1-\beta^k_{ijO}) \frac{\zeta^k_{ij}}{\zeta^k_{ijV}} \frac{\zeta^k_{ij}}{\zeta^k_{ijO}} \left(\frac{\zeta^k_{ij}}{\zeta^k_{ijO}}\beta^k_{ijO} - \frac{\zeta^k_{ij}}{\zeta^k_{ijV}}\beta^k_{ijV} \right) \right].$$

From the proof of Lemma 2(ii), we have seen that: (i) $0 < \frac{\zeta^k_{ij}}{\zeta^k_{ijO}}\beta^k_{ijO} < \frac{\zeta^k_{ij}}{\zeta^k_{ijV}}\beta^k_{ijV} < 1$, and: (ii) $0 < \frac{\zeta^k_{ij}}{\zeta^k_{ijV}}(1-\beta^k_{ijV}) < \frac{\zeta^k_{ij}}{\zeta^k_{ijO}}(1-\beta^k_{ijO}) < 1$; together, these imply that: $\frac{\zeta^k_{ij}}{\zeta^k_{ijV}}(1-\beta^k_{ijV})(1-\frac{\zeta^k_{ij}}{\zeta^k_{ijV}}\beta^k_{ijV}) < \frac{\zeta^k_{ij}}{\zeta^k_{ijO}}(1-\beta^k_{ijO})(1-\frac{\zeta^k_{ij}}{\zeta^k_{ijO}}\beta^k_{ijO})$. The terms in the large square brackets of the last expression above for $\frac{d}{d\mu^k_{hij}}\log(b^k_{ijV}/b^k_{ijO})$ therefore collectively have a negative sign, from which it follows that the intrafirm trade share by value is decreasing in μ^k_{hij}.

Turning to the log derivative with respect to μ^k_{xij}:

$$\frac{d}{d\mu^k_{xij}} \log \frac{b^k_{ijV}}{b^k_{ijO}} = \frac{1}{b^k_{ijV}} \left(1 - \frac{\zeta^k_{ij}}{\zeta^k_{ijV}}(1-\beta^k_{ijV}) + (1-\mu^k_{xij})(1-\beta^k_{ijV}) \frac{d}{d\mu^k_{xij}} \frac{\zeta^k_{ij}}{\zeta^k_{ijV}} \right)$$
$$- \frac{1}{b^k_{ijO}} \left(1 - \frac{\zeta^k_{ij}}{\zeta^k_{ijO}}(1-\beta^k_{ijO}) + (1-\mu^k_{xij})(1-\beta^k_{ijO}) \frac{d}{d\mu^k_{xij}} \frac{\zeta^k_{ij}}{\zeta^k_{ijO}} \right).$$

Once again, we use the quotient rule to evaluate $\frac{d}{d\mu^k_{xij}}\frac{\zeta^k_{ij}}{\zeta^k_{ij\chi}}$ for $\chi \in \{V, O\}$; after some algebra, we arrive at:

$$\frac{d}{d\mu^k_{xij}} \log \frac{b^k_{ijV}}{b^k_{ijO}} \;=\; g\left(\frac{\zeta^k_{ij}}{\zeta^k_{ijV}}(1-\beta^k_{ijV})\right) - g\left(\frac{\zeta^k_{ij}}{\zeta^k_{ijO}}(1-\beta^k_{ijO})\right),$$

where: $g(y) = \frac{1-y}{\mu^k_{xij}+(1-\mu^k_{xij})y}\left[1+\frac{\rho^k(1-\alpha^k)(1-\mu^k_{xij})}{\zeta^k_{ij}}y\right]$. Straightforward differentiation yields:

$$g'(y) = \frac{1}{(\mu^k_{xij}+(1-\mu^k_{xij})y)^2}\left[-1+\frac{\rho^k(1-\alpha^k)(1-\mu^k_{xij})}{\zeta^k_{ij}}(\mu^k_{xij}(1-y)^2-y^2)\right],$$

where notice that:

$$\begin{aligned}
& -\zeta^k_{ij}+\rho^k(1-\alpha^k)(1-\mu^k_{xij})(\mu^k_{xij}(1-y)^2-y^2) \\
< & -\zeta^k_{ij}+\rho^k(1-\alpha^k)(1-\mu^k_{xij})\mu^k_{xij}(1-y)^2 \\
= & -1+\rho^k\alpha^k(1-\mu^k_{hij})+\rho^k(1-\alpha^k)\left((1-\mu^k_{xij})+\mu^k_{xij}(1-\mu^k_{xij})(1-y)^2\right) \\
< & -1+\rho^k\alpha^k(1-\mu^k_{hij})+\rho^k(1-\alpha^k), \text{ for } y\in(0,1).
\end{aligned}$$

Since $\rho^k, \alpha^k, \mu^k_{hij} \in (0,1)$, we have: $-1+\rho^k\alpha^k(1-\mu^k_{hij})+\rho^k(1-\alpha^k) < -1+\rho^k < 0$. It follows that $g'(y) < 0$ for all $y \in (0,1)$. Given that: $0 < \frac{\zeta^k_{ij}}{\zeta^k_{ijV}}(1-\beta^k_{ijV}) < \frac{\zeta^k_{ij}}{\zeta^k_{ijO}}(1-\beta^k_{ijO}) < 1$, this implies: $g(\frac{\zeta^k_{ij}}{\zeta^k_{ijV}}(1-\beta^k_{ijV})) > g(\frac{\zeta^k_{ij}}{\zeta^k_{ijO}}(1-\beta^k_{ijO}))$, so that $\frac{d}{d\mu^k_{xij}}(b^k_{ijV}/b^k_{ijO}) > 0$. The intrafirm trade share by value is therefore increasing in μ^k_{xij}.

Behavior of intrafirm trade share by value with respect to β^k_{ijO}: We show that the expression in (B.2) is decreasing in β^k_{ijO}, subject to a sufficient condition similar to that seen earlier in the comparative static with respect to α^k (i.e., λ_i sufficiently large).

In the proof of Lemma 2(iii), we established that B^k_{ijV}/B^k_{ijO} is decreasing in β^k_{ijO}, and wrote down an explicit expression for $\frac{d}{d\beta^k_{ijO}}\log(B^k_{ijV}/B^k_{ijO})$ in equation (A.17). We next work out an expression for $\frac{d}{d\beta^k_{ijO}}\log(b^k_{ijV}/b^k_{ijO})$:

$$\frac{d}{d\beta^k_{ijO}}\log\frac{b^k_{ijV}}{b^k_{ijO}} = \frac{(1-\mu^k_{xij})\zeta^k_{ij}}{b^k_{ijV}}\frac{d}{d\beta^k_{ijO}}\frac{1-\beta^k_{ijV}}{\zeta^k_{ijV}} - \frac{(1-\mu^k_{xij})\zeta^k_{ij}}{b^k_{ijO}}\frac{d}{d\beta^k_{ijO}}\frac{1-\beta^k_{ijO}}{\zeta^k_{ijO}}.$$

Bearing in mind that $\beta^k_{ijV} = \delta^k_{ij}+\beta^k_{ijO}(1-\delta^k_{ij})$, straightforward differentiation and some simplification yields: $\frac{d}{d\beta^k_{ijO}}\frac{1-\beta^k_{ijV}}{\zeta^k_{ijV}} = -\frac{1-\delta^k_{ij}}{(\zeta^k_{ijV})^2}(1-\rho^k\alpha^k(1-\mu^k_{hij}))$ and $\frac{d}{d\beta^k_{ijO}}\frac{1-\beta^k_{ijO}}{\zeta^k_{ijO}} = -\frac{1}{(\zeta^k_{ijO})^2}(1-\rho^k\alpha^k(1-\mu^k_{hij}))$, which we can substitute into the above expression for $\frac{d}{d\beta^k_{ijO}}\log(b^k_{ijV}/b^k_{ijO})$. Bringing this together with (A.17), one obtains the following expression for $\frac{d}{d\beta^k_{ijO}}(\frac{\theta^k}{1-\lambda_i}\log(B^k_{ijV}/B^k_{ijO})+\log(b^k_{ijV}/b^k_{ijO}))$:

$$\begin{aligned}
&\frac{\theta^k}{1-\lambda_i}\frac{\delta^k_{ij}\alpha^k(1-\mu^k_{hij})}{1-\alpha^k}\left[\frac{\zeta^k_{ij}}{\zeta^k_{ijV}\zeta^k_{ijO}}(1-\rho^k\alpha^k(1-\mu^k_{hij}))\left(1-\frac{(1-\alpha^k)(1-\mu^k_{xij})}{\alpha^k(1-\mu^k_{hij})}\right)-\frac{1}{\beta^k_{ijV}\beta^k_{ijO}}\right] \\
&\quad+\frac{(1-\mu^k_{xij})\zeta^k_{ij}}{b^k_{ijV}b^k_{ijO}}(1-\rho^k\alpha^k(1-\mu^k_{hij}))\left(-\frac{1-\delta^k_{ij}}{(\zeta^k_{ijV})^2}b^k_{ijO}+\frac{1}{(\zeta^k_{ijO})^2}b^k_{ijV}\right).
\end{aligned}$$

We know from the proof of Lemma 2(iii) that the first line of this last equation has a negative sign. On the other hand, the second line cannot be signed explicitly; this can be positive, and as a result, the intrafirm trade share by value can be increasing in β^k_{ijO}, although the parameterizations under which this occurs are relatively extreme.[63] That said, if λ_i is large and approaches its maximum value of 1, the negative term in the first line will increase in magnitude and tend to $-\infty$ in the limit, whereas the term in the second line will remain finite and bounded. The condition that λ_i be large is thus sufficient to ensure that $\frac{d}{d\beta^k_{ijO}}(\frac{\theta^k}{1-\lambda_i}\log(B^k_{ijV}/B^k_{ijO})+$

[63]For example, consider $\rho^k = 0.75$, $\alpha^k = 0.3$, $\mu^k_{hij} = 0.99$, $\mu^k_{xij} = 0.01$, $\delta^k_{ij} = 0.99$, $\theta^k = 4$, and $\lambda_i = 0$. The intrafirm trade share by value is equal to 1.348×10^{-9} at $\beta^k_{ijO} = 0.5$, while it increases to 2.547×10^{-9} at $\beta^k_{ijO} = 0.9$.

$\log(b^k_{ijV}/b^k_{ijO})) < 0$, i.e., that the comparative static behavior of (B^k_{ijV}/B^k_{ijO}) on the extensive margin will dominate any shifts in the (b^k_{ijV}/b^k_{ijO}) intensive margin term. (Note again that there are alternative conditions that one could introduce – for example, α^k large, or μ^k_{xij} large – which would also yield this comparative static.)

Behavior of intrafirm trade share by value with respect to δ^k_{ij}: Recall that we have an expression for $\frac{d}{d\delta^k_{ij}}\log(B^k_{ijV}/B^k_{ijO})$ from equation (A.18). On the other hand:

$$\frac{d}{d\delta^k_{ij}}\log\frac{b^k_{ijV}}{b^k_{ijO}} = \frac{(1-\mu^k_{xij})\zeta^k_{ij}}{b^k_{ijV}}\frac{d}{d\delta^k_{ij}}\frac{1-\beta^k_{ijV}}{\zeta^k_{ijV}}.$$

Since $1-\beta^k_{ijV} = (1-\beta^k_{ijO})(1-\delta^k_{ij})$, one can then show that: $\frac{d}{d\delta^k_{ij}}\frac{1-\beta^k_{ijV}}{\zeta^k_{ijV}} = -\frac{1-\beta^k_{ijO}}{(\zeta^k_{ijV})^2}(1-\rho^k\alpha^k(1-\mu^k_{hij}))$; notice in particular that this implies: $\frac{d}{d\delta^k_{ij}}\log(b^k_{ijV}/b^k_{ijO}) < 0$. We thus have:

$$\frac{d}{d\delta^k_{ij}}\left(\frac{\theta^k}{1-\lambda_i}\log\frac{B^k_{ijV}}{B^k_{ijO}} + \log\frac{b^k_{ijV}}{b^k_{ijO}}\right) = \frac{\theta^k}{1-\lambda_i}\frac{1}{1-\alpha^k}\Psi - \frac{(1-\mu^k_{xij})\zeta^k_{ij}}{b^k_{ijV}}\frac{1-\beta^k_{ijO}}{(\zeta^k_{ijV})^2}(1-\rho^k\alpha^k(1-\mu^k_{hij})),$$

where Ψ is as defined in the proof of Lemma 2(iv).

Moreover, the second derivative $\frac{d^2}{d(\delta^k_{ij})^2}(\frac{\theta^k}{1-\lambda_i}\log(B^k_{ijV}/B^k_{ijO}) + \log(b^k_{ijV}/b^k_{ijO}))$ is equal to:

$$\begin{aligned}
&\frac{\theta^k}{1-\lambda_i}\frac{1}{1-\alpha^k}\frac{d\Psi}{d\delta^k_{ij}} - (1-\mu^k_{xij})\zeta^k_{ij}(1-\beta^k_{ijO})(1-\rho^k\alpha^k(1-\mu^k_{hij}))\frac{d}{d\delta^k_{ij}}\frac{1/(\zeta^k_{ijV})^2}{b^k_{ijV}} \\
&= \frac{\theta^k}{1-\lambda_i}\frac{1}{1-\alpha^k}\frac{d\Psi}{d\delta^k_{ij}} + \frac{1}{(\zeta^k_{ijV})^2}\left[2(1-\beta^k_{ijO})\rho^k(-\alpha^k(1-\mu^k_{hij}) + (1-\alpha^k)(1-\mu^k_{xij}))(1-\phi^k_{ij})\frac{1-\rho^k\alpha^k(1-\mu^k_{hij})}{1-\delta^k_{ij}}\right. \\
&\qquad \left. -(1-\phi^k_{ij})^2\left(\frac{1-\rho^k\alpha^k(1-\mu^k_{hij})}{1-\delta^k_{ij}}\right)^2\right]. \qquad \text{(B.5)}
\end{aligned}$$

where recall that $\phi^k_{ij} \equiv \frac{\mu^k_{xij}}{b^k_{ijV}}$. Separately, the expression for $\frac{d\Psi}{d\delta^k_{ij}}$ in equation (A.19) yields:

$$\begin{aligned}
\frac{\theta^k}{1-\lambda_i}\frac{1}{1-\alpha^k}\frac{d\Psi}{d\delta^k_{ij}} &< -\frac{\theta^k}{1-\lambda_i}\frac{1}{1-\alpha^k}\rho^k\frac{\zeta^k_{ij}}{(\zeta^k_{ijV})^2}(1-\beta^k_{ijO})^2\left(-\alpha^k(1-\mu^k_{hij}) + (1-\alpha^k)(1-\mu^k_{xij})\right)^2 \\
&< -\frac{\zeta^k_{ij}}{1-\rho^k}(\rho^k)^2\frac{1}{(\zeta^k_{ijV})^2}(1-\beta^k_{ijO})^2\left(-\alpha^k(1-\mu^k_{hij}) + (1-\alpha^k)(1-\mu^k_{xij})\right)^2 \\
&< -(\rho^k)^2\frac{1}{(\zeta^k_{ijV})^2}(1-\beta^k_{ijO})^2\left(-\alpha^k(1-\mu^k_{hij}) + (1-\alpha^k)(1-\mu^k_{xij})\right)^2.
\end{aligned}$$

Note that the first inequality comes from discarding the second and third negative terms in equation (A.19); the second inequality follows from the parameter restriction, $\theta^k > \frac{\rho^k}{1-\rho^k}(1-\alpha^k)$, as well as the fact that $\lambda_i \in (0,1)$; while the third inequality is a consequence of $\zeta^k_{ij} > 1-\rho^k$. Plugging this inequality into (B.5), the second derivative of interest satisfies the inequality:

$$\begin{aligned}
&< -\frac{1}{(\zeta^k_{ijV})^2}\left[\rho^k(1-\beta^k_{ijO})(-\alpha^k(1-\mu^k_{hij}) + (1-\alpha^k)(1-\mu^k_{xij})) - (1-\phi^k_{ij})\left(\frac{1-\rho^k\alpha^k(1-\mu^k_{hij})}{1-\delta^k_{ij}}\right)\right]^2 \\
&< 0.
\end{aligned}$$

Hence, $\frac{\theta^k}{1-\lambda_i}\log(B^k_{ijV}/B^k_{ijO}) + \log(b^k_{ijV}/b^k_{ijO})$ is concave with respect to δ^k_{ij}. Also, $\frac{d}{d\delta^k_{ij}}(\frac{\theta^k}{1-\lambda_i}\log(B^k_{ijV}/B^k_{ijO}) + \log(b^k_{ijV}/b^k_{ijO}))$ tends to $-\infty$ as $\delta^k_{ij} \longrightarrow 1$, since from (A.19), $\frac{d\Psi}{d\delta^k_{ij}}$ tends to $-\infty$ as $\delta^k_{ij} \longrightarrow 1$.

The behavior of $\frac{\theta^k}{1-\lambda_i}\log(B^k_{ijV}/B^k_{ijO}) + \log(b^k_{ijV}/b^k_{ijO})$ with respect to δ^k_{ij} is thus qualitatively similar to that of $\log(B^k_{ijV}/B^k_{ijO})$ in the proof of Lemma 2(iv): If $\frac{d}{d\delta^k_{ij}}(\frac{\theta^k}{1-\lambda_i}\log(B^k_{ijV}/B^k_{ijO}) + \log(b^k_{ijV}/b^k_{ijO})) \leq 0$ at $\delta^k_{ij} = 0$, we would have $\frac{\theta^k}{1-\lambda_i}\log(B^k_{ijV}/B^k_{ijO}) + \log(b^k_{ijV}/b^k_{ijO})$ and hence the intrafirm trade share by value being a monotonic decreasing function in δ_{ij}^k over the entire range of $\delta_{ij}^k \in (0,1)$. On the other hand, if $\frac{d}{d\delta_{ij}^k}(\frac{\theta^k}{1-\lambda_i}\log(B_{ijV}^k/B_{ijO}^k) + \log(b_{ijV}^k/b_{ijO}^k)) > 0$ at $\delta_{ij}^k = 0$, the intrafirm trade share by value will be increasing in δ_{ij}^k from $\delta_{ij}^k = 0$ up to some cutoff value of δ_{ij}^k, while it will be decreasing in δ_{ij}^k thereafter. Putting together these two possibilities, the intrafirm trade share by value is decreasing in δ_{ij}^k for sufficiently high values of δ_{ij}^k. As was the case in Lemma 2(iv), our numerical exploration shows that this relationship between the intrafirm trade share by value and δ_{ij}^k tends to be a monotonic decreasing one over the entire range $\delta_{ij}^k \in (0,1)$, except when the underlying model parameters take on extreme values.

B.3 Estimation Strategy Details

Functional forms: In the empirical implementation, we specify α^k, μ_{hij}^k, μ_{xij}^k, β_{ijO}^k, and δ_{ij}^k to be standard logistic transformations of polynomial functions of country and industry observables. Recall that the second-order polynomials associated with α^k, μ_{hij}^k, μ_{xij}^k, β_{ijO}^k, and δ_{ij}^k are respectively $\mathbf{a}(\cdot)$, $\mathbf{h}(\cdot)$, $\mathbf{x}(\cdot)$, $\mathbf{b}(\cdot)$, and $\mathbf{d}(\cdot)$. We spell out the polynomial functions we use in our baseline specification in full below:

$$\mathbf{a}(\cdot) = \gamma_{\alpha 0} + \gamma_{\alpha 1}\log(K/L)^k + \gamma_{\alpha 2}\left(\log(K/L)^k\right)^2 \tag{B.6}$$

$$\begin{aligned}\mathbf{h}(\cdot) = \gamma_{\mu h0} &+ \gamma_{\mu h1}\left(\text{HQContractibility}^k\right) + \gamma_{\mu h2}\left(\text{HQContractibility}^k\right)^2 \\ &+ \gamma_{\mu h3}\text{ROL}_i + \gamma_{\mu h4}\left(\text{ROL}_i\right)^2 + \gamma_{\mu h5}\text{ROL}_j + \gamma_{\mu h6}\left(\text{ROL}_j\right)^2 \\ &+ \gamma_{\mu h7}\left(\text{HQContractibility}^k\right)\times\text{ROL}_i + \gamma_{\mu h8}\left(\text{HQContractibility}^k\right)\times\text{ROL}_j,\end{aligned} \tag{B.7}$$

$$\begin{aligned}\mathbf{x}(\cdot) = \gamma_{\mu x0} &+ \gamma_{\mu x1}\left(\text{SSContractibility}^k\right) + \gamma_{\mu x2}\left(\text{SSContractibility}^k\right)^2 \\ &+ \gamma_{\mu x3}\text{ROL}_i + \gamma_{\mu x4}\left(\text{ROL}_i\right)^2 + \gamma_{\mu x5}\text{ROL}_j + \gamma_{\mu x6}\left(\text{ROL}_j\right)^2 \\ &+ \gamma_{\mu x7}\left(\text{SSContractibility}^k\right)\times\text{ROL}_i + \gamma_{\mu x8}\left(\text{SSContractibility}^k\right)\times\text{ROL}_j,\end{aligned} \tag{B.8}$$

$$\mathbf{b}(\cdot) = \gamma_{\beta 0} + \gamma_{\beta 1}\text{Markup}^k + \gamma_{\beta 2}\left(\text{Markup}^k\right)^2 \tag{B.9}$$

$$\begin{aligned}\mathbf{d}(\cdot) = \gamma_{\delta 0} &+ \gamma_{\delta 1}\left(\text{Specificity}^k\right) + \gamma_{\delta 2}\left(\text{Specificity}^k\right)^2 \\ &+ \gamma_{\delta 3}\text{ROL}_i + \gamma_{\delta 4}\left(\text{ROL}_i\right)^2 + \gamma_{\delta 5}\text{ROL}_j + \gamma_{\delta 6}\left(\text{ROL}_j\right)^2 \\ &+ \gamma_{\delta 7}\left(\text{Specificity}^k\right)\times\text{ROL}_i + \gamma_{\delta 8}\left(\text{Specificity}^k\right)\times\text{ROL}_j + \gamma_{\delta 9}\text{BIT}_{ij}.\end{aligned} \tag{B.10}$$

In particular, $\mathbf{x}(\cdot)$, $\mathbf{h}(\cdot)$, and $\mathbf{d}(\cdot)$ include full second-order polynomials of an industry-level variable (indexed by k), the rule of law of the source country (i), and the rule of law of the destination country (j), except that we omit the $\text{ROL}_i \times \text{ROL}_j$ term. This is because our intrafirm trade data from the US Census Bureau's Related Party Database are not fully balanced at the bilateral level; instead, the US is either the importing or exporting country in each observation. With this data constraint, it turns out that the $\text{ROL}_i \times \text{ROL}_j$ interaction can be written as a linear combination of ROL_i, ROL_j, and a constant. To see this, let r_{US} denote the ROL value of the US. We then have: $[\text{ROL}_i \times \text{ROL}_j] = r_{US}\left([\text{ROL}_i] + [\text{ROL}_j] - r_{US}[\mathbf{1}]\right)$, where $[\cdot]$ refers to the column vector that stacks the observations of the corresponding variable, and $[\mathbf{1}]$ is a column vector of one's. The inclusion of $\text{ROL}_i \times \text{ROL}_j$ would therefore result in a collinearity issue with ROL_i, ROL_j, or the constant term. Note that if one had access to richer data on intrafirm trade shares among country pairs that exclude the US, one could then (in principle) reincorporate the $\text{ROL}_i \times \text{ROL}_j$ interaction in the empirical specification.

Pinning down ρ^k: Soderbery (2015) computes import demand elasticities for "goods", defined as products at the HS10 level; the variation exploited for estimation within a code is that across "varieties" defined by country of origin, based on the Armington (1969) assumption. In our context, we treat each input variety l^k as a "good" (in line with the detailed nature of HS10 codes); following Soderbery (2015), we view input varieties from different source countries as distinct "varieties".

We derive the expression implied by our model for the (expected) value of trade flows of a given input variety l^k that originates from country i and is purchased in country j, which we denote by $t^k_{ij}(l^k)$. This can be mapped naturally to the notion of product-level trade flows in Soderbery (2015). Specifically, $t^k_{ij}(l^k)$ is given by:

$$t^k_{ij}(l^k) = N_j \int_\phi \mathbb{E}_{\tilde{z}} \left[\sum_{\chi \in \{V,O\}} c^k_{ij\chi} \left(\mu^k_{xij} x^k_{cj}(l^k;\phi) + (1-\mu^k_{xij}) x^k_{nj}(l^k;\phi) \right) \right] \, dG_j(\phi).$$

Recall that in Section 3.2 in the main paper, we aggregated across input varieties to get an expression for trade flows at the country pair by industry by organizational mode level. In contrast to that earlier computation, we are now aggregating the trade flow values within an input variety l^k across the core productivity ϕ of country-j firms purchasing l^k from country i, as well as over organizational mode χ. Note that the expectation within the integrand, $\mathbb{E}_{\tilde{z}}[\cdot]$, is taken over the distribution of the supplier draws $Z^k_{ij\chi}(l^k;\phi)$ conditional on (i,χ) being in fact the optimal sourcing mode that solves the decision problem in (17).

Drawing on the prior trade flow derivations in Appendix A.2, we now substitute in the expressions for $x^k_{cj}(l^k;\phi)$ and $x^k_{nj}(l^k;\phi)$ from (12); simplifying extensively, we obtain:

$$\begin{aligned} t^k_{ij}(l^k) = & \left[N_j \int \left(\frac{(1-\alpha)\rho\eta^k R_j(\phi)}{(X^k_j(\phi))^{\rho^k}} \right)^{\frac{1}{1-\rho^k}} dG_j(\phi) \right] \left(\frac{\alpha^k}{s_j} \right)^{\frac{\rho^k \alpha^k}{1-\rho^k}} \left(1-\alpha^k \right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}+1} \\ & \times \sum_{\chi \in \{V,O\}} \left(\mu^k_{xij} + (1-\mu^k_{xij})(1-\beta^k_{ij\chi}) \frac{\zeta^k_{ij}}{\zeta^k_{ij\chi}} \right) \left(B^k_{ij\chi} \right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} \mathbb{E}_{\tilde{z}} \left[\left(\frac{Z^k_{ij\chi}(l^k)}{d^k_{ij} w_i} \right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} \right], \end{aligned}$$

where we have removed the dependence of the $Z^k_{ij\chi}(l^k)$'s on ϕ, since the distribution of country-j firms' core productivity is independent from that of the nested-Fréchet supplier draws.

Denote the expected unit labor supplier costs to be: $\bar{c}^k_{ij\chi} = d^k_{ij} w_i / \mathbb{E}_{\tilde{z}}[Z^k_{ij\chi}(l^k)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}]$. With this definition:

$$\left(\bar{c}^k_{ij\chi} \right)^{-\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} = \mathbb{E}_{\tilde{z}} \left[\left(\frac{Z^k_{ij\chi}(l^k;\phi)}{d^k_{ij} w_i} \right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} \right] = \mathbb{E}_{\tilde{z}} \left[\left(c^k_{ij\chi} \right)^{-\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} \right],$$

so that $\bar{c}^k_{ij\chi}$ is a generalized harmonic mean of the unit labor supplier costs, $c^k_{ij\chi} = d^k_{ij} w_i / Z^k_{ij\chi}(l^k)$, taken over the distribution of the $Z^k_{ij\chi}(l^k)$'s. Plugging this into the expression for $t^k_{ij}(l^k)$ yields:

$$t^k_{ij}(l^k) \propto \sum_{\chi \in \{V,O\}} \left(\mu^k_{xij} + (1-\mu^k_{xij})(1-\beta^k_{ij\chi}) \frac{\zeta^k_{ij}}{\zeta^k_{ij\chi}} \right) \left(B^k_{ij\chi} \right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} \left(\bar{c}^k_{ij\chi} \right)^{-\frac{\rho^k(1-\alpha^k)}{1-\rho^k}},$$

where '$\propto$' denotes equality up to multiplicative terms that do not depend on χ. The response of the value of trade flows to a shock to prices (which in our setting increase linearly in unit costs) is therefore equal to $-\frac{\rho^k(1-\alpha^k)}{1-\rho^k}$, if this is a shock that uniformly hits unit costs for both organizational modes. This maps in the Soderbery (2015) setting to $1-\sigma^k$, where σ^k is the demand elasticity (the response of quantity demanded to a shock in prices). With information on the demand elasticities σ^k from Soderbery (2015) and estimated values for α^k from its parameterization via $\mathbf{a}(\cdot)$, one can then infer an implied value for ρ^k.

B.4 Data and Sample Details

Intrafirm trade share: From the US Census Bureau's Related Party Trade Database; constructed as the value of related party imports (respectively, exports) in a given NAICS 3-digit industry, expressed as a share of the total value of related and non-related party imports (exports) in that same industry code. This is computed for each year between 2001 and 2005, with a simple average then taken across years to smooth out idiosyncratic noise from any given year. The US Census Bureau considers imports to be between related parties if they are linked via a direct or indirect ownership stake that is at least 5%. On the other hand, it deems exports to be between related parties if there is a direct or indirect ownership link of at least 10%. While the thresholds are different, Nunn and Trefler (2008) argue that both of these thresholds are sufficiently high to be deemed substantive in terms of control rights conferred. The 21 NAICS 3-digit manufacturing industries are listed in Table B.1.

For estimation, our country sample comprises the top 50 sources of imports for the US; this is computed by ranking each economy's total value of related and non-related party manufacturing imports between 2001-2005. We exclude Iraq, Saudi Arabia, and Venezuela, which are predominantly oil exporters, as well as Hong Kong, due to its entrepot status vis-à-vis China.

Capital intensity: From the NBER-CES Manufacturing Industry dataset (Becker et al., 2021). For each NAICS 3-digit industry, real capital stock and employment are respectively summed up across 2001-2005, from which the log real capital stock per worker, $\ln(K/L)^k$, is computed.

Specificity: From Rauch (1999). The Rauch measure is originally coded up at the SITC level; we map these to NAICS categories using concordance weights based on the total value of US imports by NAICS-SITC cells for 1989-2006 observed in Feenstra et al. (2002). For each NAICS 3-digit industry, we compute Specificity^k as the share of SITC 4-digit codes within the NAICS industry that are classified as differentiated (as opposed to reference-priced or exchange-traded). We use the "conservative" coding in Rauch (1999); this yields a slightly lower share of codes deemed to be reference-priced or exchange-traded compared to the "liberal" coding.

Contractibility: From the specificity measure just described and the 1997 US Input-Output Tables. The latter provide a NAICS-IO concordance, which we first use to construct direct requirements coefficients for each NAICS 6-digit output industry in manufacturing, focusing on its use of manufacturing inputs (since these are the inputs for which we have data to rank them by physical capital intensity). The mapping from NAICS 6-digit industries in manufacturing to IO 6-digit industry codes is many-to-one: Each NAICS 6-digit industry in manufacturing is associated with a unique IO industry code, but there can be multiple NAICS 6-digit industries associated with a given IO code. For a given NAICS 6-digit output industry in manufacturing, we first take the direct requirements coefficients for the unique IO output industry it is associated with. To compute this output industry's use of inputs from a given NAICS 6-digit input industry, we adopt an equal apportionment rule, i.e., we divide the direct requirements coefficients by the number of NAICS 6-digit input industries associated in turn with each IO input industry code.

We next merge specificity values at the NAICS 6-digit level into this concordance. (If the specificity value is initially missing for a given NAICS 6-digit industry, we replace it with the share of codes that are classified as differentiated at the 5-digit level; if that is still missing, we successively replace it with the shares at the 4- and 3-digit levels as necessary.) We then partition the NAICS 6-digit industries into those with an above-median (respectively, below-median) value of log real physical capital per worker, designating the former (respectively, latter) as inputs likely to be provided by firm headquarters (respectively, suppliers). Using the direct requirements coefficients, we then compute the share of inputs (by value) used in NAICS 3-digit industry k that are traded on an open exchange or reference-priced (i.e., one minus the share that are classified as differentiated) separately for each of these subsets of inputs; this yields the "HQContractibility^k" and "SSContractibility^k" measures.

Markup: From De Loecker et al. (2016). We use the data provided in the second column of their Table 6, which is the mean markup in each industry. Table B.1 provides the mapping between the 11 industries under the Indian National Industry Classification (NIC) used in De Loecker et al. (2016) and NAICS 3-digit industries. For the five NAICS 3-digit industries that cannot be directly mapped to the NIC, we assign the average markup value across all NIC industries (2.7).

NAICS3	NAICS3 Description	NIC	NIC Description
311	Food Manufacturing	15	Food products and beverages
312	Beverage and Tobacco Product Manufacturing	15	Food products and beverages
313	Textile Mills	17	Textiles, apparel
314	Textile Product Mills	17	Textiles, apparel
315	Apparel Manufacturing	17	Textiles, apparel
316	Leather and Allied Product Manufacturing	17	Textiles, apparel
321	Wood Product Manufacturing	-	-
322	Paper Manufacturing	21	Paper and paper products
323	Printing and Related Support Activities	-	-
324	Petroleum and Coal Products Manufacturing	-	-
325	Chemical Manufacturing	24	Chemicals
326	Plastics and Rubber Products Manufacturing	25	Rubber and plastic
327	Nonmetallic Mineral Product Manufacturing	26	Nonmetallic mineral products
331	Primary Metal Manufacturing	27	Basic metals
332	Fabricated Metal Product Manufacturing	28	Fabricated metal products
333	Machinery Manufacturing	29	Machinery and equipment
334	Computer and Electronic Product Manufacturing	31	Electrical machinery and communications
335	Electrical Equipment Appliance and Component Manufacturing	31	Electrical machinery and communications
336	Transportation Equipment Manufacturing	34	Motor vehicles, trailers
337	Furniture and Related Product Manufacturing	-	-
339	Miscellaneous Manufacturing	-	-

Table B.1: Mapping NAICS3 to NIC

Import demand elasticities: From Soderbery (2015). These are originally estimated at the HS 10-digit level. We map these to NAICS industries by using HS-NAICS concordance weights computed from the value of US imports between 1989-2006 from Feenstra et al. (2002), and taking a weighted-average of the import demand elasticities over all constituent HS 10-digit codes for a given NAICS 3-digit industry. (When an elasticity is missing for a HS 10-digit code, we fill it in using the weighted-average of available HS10 codes at the HS9 level; if that is still missing, we do so as necessary to replace it with an import demand elasticity at the next higher level of aggregation, all the way up till the HS2 level.) We then map the demand elasticity computed in this way to the ρ^k's in our model, as detailed previously in Appendix B.3.

Rule of law: From the World Bank World Governance Indicators. We average the ROL index over 2002-2013. The original index lies between -2.5 and 2.5; we linearly re-scale this so that it lies between 0 and 1.

Country income groups: From the World Bank. The classification of countries into lower-middle, upper-middle, and high income groups follows the World Bank's definition in 2004. This is used for the estimation of a common λ_i correlation parameter within country income groups.

Bilateral investment treaties: From the World Bank's Database of Bilateral Investment Treaties (BITs). In our sample, two countries are considered to have an active BIT if the treaty entered into force on or before the year 2005.

Data for counterfactuals: From the OECD's Inter-Country Input-Output Tables (ICIO, 2018 version). We use the data for the year 2005. The ICIO entries provide information on the value of bilateral trade flows by industry. The ICIO Tables classify industries at the ISIC Rev 4 level, which we map into NAICS 3-digit industries by matching the industry descriptions. If an ISIC code maps into multiple NAICS 3-digit industries, we split the ICIO entries for input purchases into the corresponding NAICS entries in proportion to the value added of the two NAICS industries (obtained from the NBER-CES dataset).[64]

The data on country-level expenditure also come from the ICIO Tables. We use the sum of the six "final demand" columns of a country j as expenditure, E_j, in the counterfactual exercises. The final demand columns are: "Household final consumption expenditure", "Non-profit institutions serving households", "General government final consumption", "Gross fixed capital formation", "Change in inventories and valuables", and "Direct purchases abroad".

[64] For example, D10T12 maps into both 311 and 312. In the NBER-CES dataset, NAICS 311 is roughly 3 times larger than 312 by value added, and therefore, we assign 74% of the trade flows in the ICIO code to NAICS 311 and 26% to 312.

NAICS3	NAICS3 Description	ICIO	Weights
311	Food Manufacturing	D10T12	0.74
312	Beverage and Tobacco Product Manufacturing	D10T12	0.26
313	Textile Mills	D13T15	0.34
314	Textile Product Mills	D13T15	0.29
315	Apparel Manufacturing	D13T15	0.32
316	Leather and Allied Product Manufacturing	D13T15	0.06
321	Wood Product Manufacturing	D16	-
322	Paper Manufacturing	D17T18	0.56
323	Printing and Related Support Activities	D17T18	0.44
324	Petroleum and Coal Products Manufacturing	D19	-
325	Chemical Manufacturing	D20T21	-
326	Plastics and Rubber Products Manufacturing	D22	-
327	Nonmetallic Mineral Product Manufacturing	D23	-
331	Primary Metal Manufacturing	D24	-
332	Fabricated Metal Product Manufacturing	D25	-
333	Machinery Manufacturing	D28	-
334	Computer and Electronic Product Manufacturing	D26	-
335	Electrical Equipment Appliance and Component Manufactur	D27	-
336	Transportation Equipment Manufacturing	D29 + D30	-
337	Furniture and Related Product Manufacturing	D31T33	0.33
339	Miscellaneous Manufacturing	D31T33	0.67

Table B.2: Industry mapping between ISIC Rev 4 and NAICS 3-digit industries

Table B.3 lists the countries in the samples we use respectively for structural estimation and the counterfactual exercises. There are six trade partners of the US that are in our sample for estimation, but not included in the OECD ICIO tables.

ARG	DEU	IND	PER
AUS	DNK	IRL	PHL
AUT	DOM*	ISR	ROW†
BEL	DZA*	ITA	RUS
BGD*	ESP	JPN	SGP
BRA	FIN	KOR	SWE
CAN	FRA	MEX	THA
CHE	GBR	MYS	TUR
CHL	GTM*	NLD	TWN
CHN	HND*	NOR	USA
COL	HUN	NZL	VNM
CRI	IDN	PAK*	ZAF

Table B.3: Countries in the quantitative exercise

Notes: The six countries with a * are only in the structural estimation; these are not in the counterfactual simulations because they are not covered in the ICIO data. "ROW" (with a †) stands for "rest-of-the-world"; this is included only in the counterfactual analysis.

B.5 Estimation: Technical Details

We use a combination of the Particle Swarm Optimization (PSO) and Levenberg-Marquardt (LM) algorithms to solve the weighted-NLLS minimization problem in (36). There are 57 parameters to estimate, these being the 20 θ^k's (after normalizing $\theta^1 = 4$), the λ_i correlation parameters for the three country income groups, and the 34 polynomial coefficients (including constant terms) in (B.6)-(B.10). Recall that we have 1,926 data points.

We start with the PSO algorithm, which does not take a stand on the initial guess and instead searches broadly across the entire parameter space. We use a population of 500 "particles", about 10 times the number of parameters to be estimated, where a "particle" is a 57-by-1 vector in the parameter space. At a given iteration, the PSO computes the objective function at the 500 "particles", finds the particle with the lowest weighted sum of squared residuals, and then moves all the particles closer to the best-performing particle by a pre-fixed step. This simple implementation could however miss the global best because all the particles move toward a single best point in each iteration. The PSO therefore also randomly redraws neighbors in each iteration to improve the ability of the algorithm not to get trapped in a population-best point. We call the PSO routine 10,000 times with randomized seeds and use the best point estimate from these calls as the starting point for the LM solver; we repeat the PSO until we cannot get an improvement in the objective function after a stall limit of 10 (ten consecutive calls of the PSO). In practice, the 10,000 randomized calls converge to similar point estimates, which provides assurance that the algorithm is not stuck with a local minima.

We switch to the gradient-based LM algorithm after 10,000 randomized PSO calls, and use the solution from the PSO as the initial guess for the LM algorithm. Compared to the PSO, a gradient-based method reduces computational load and guarantees local convergence. With a reasonably good initial guess, switching to a gradient-based method improves the efficiency of the estimation. The LM solver comes from the MINPACK library, available in the public domain. We use `1.0E-4` as the tolerance level that stops the LM algorithm. While the minimization routine does not conceptually guarantee a global minimum, we show below that the objective function is well-behaved in the neighborhood of our final point estimates.

B.6 Baseline Estimation Results: Details

Table B.4 below reports the weighted-NLLS estimation results in full. Here, we list the point estimates obtained for each element of the parameter vector, Θ, together with 95% confidence intervals computed based on 200 bootstrap samples. (We report the empirical confidence intervals based on the 2.5th to 97.5th percentile values across the bootstrap samples.)

name	est.	95% CI
θ_{01}	4.000	-
θ_{02}	16.923***	[4.37, 52.87]
θ_{03}	4.939***	[3.79, 7.67]
θ_{04}	5.818***	[3.03, 12.94]
θ_{05}	6.330***	[4.73, 18.76]
θ_{06}	8.752***	[3.42, 21.61]
θ_{07}	7.342***	[3.15, 28.94]

(a) θ^1 to θ^7

name	est.	95% CI
θ_{08}	8.455***	[1.86, 18.00]
θ_{09}	11.804***	[4.87, 23.96]
θ_{10}	9.168***	[6.21, 23.16]
θ_{11}	19.860***	[11.48, 35.74]
θ_{12}	6.762***	[5.93, 26.56]
θ_{13}	25.483***	[13.11, 35.81]
θ_{14}	23.719***	[15.07, 32.19]

(b) θ^8 to θ^{14}

name	est.	95% CI
θ_{15}	7.524***	[2.35, 20.36]
θ_{16}	13.945***	[5.14, 28.92]
θ_{17}	13.572***	[4.12, 25.04]
θ_{18}	1.430***	[1.37, 19.89]
θ_{19}	18.245***	[5.94, 24.84]
θ_{20}	6.676***	[0.77, 15.34]
θ_{21}	5.447***	[2.11, 12.12]

(c) θ^{15} to θ^{21}

name	est.	95% CI
λ_{01}	0.944***	[0.63, 0.99]
λ_{02}	0.923***	[0.43, 0.98]
λ_{03}	0.852***	[0.13, 0.96]

(d) λ_i's

name	est.	95% CI
$\gamma_{\alpha 0}$	-0.598**	[-7.05, -0.14]
$\gamma_{\alpha 1}$	0.289***	[0.04, 2.92]
$\gamma_{\alpha 2}$	-0.077**	[-1.40, -0.01]

(e) α^k: $\mathbf{a}(\cdot)$

name	est.	95% CI
$\gamma_{\mu h0}$	-16.364***	[-22.19, -8.00]
$\gamma_{\mu h1}$	9.199**	[2.61, 16.46]
$\gamma_{\mu h2}$	-0.409	[-4.77, 4.22]
$\gamma_{\mu h3}$	-6.194	[-21.98, 12.54]
$\gamma_{\mu h4}$	1.403	[-16.29, 14.71]
$\gamma_{\mu h5}$	10.097***	[6.82, 15.64]
$\gamma_{\mu h6}$	17.813***	[8.12, 22.14]
$\gamma_{\mu h7}$	0.414	[-1.43, 5.26]
$\gamma_{\mu h8}$	-10.291**	[-19.91, -1.42]

(f) μ^k_{hij}: $\mathbf{h}(\cdot)$

name	est.	95% CI
$\gamma_{\mu x0}$	-8.212***	[-15.32, -2.03]
$\gamma_{\mu x1}$	4.068	[-11.51, 17.65]
$\gamma_{\mu x2}$	-9.209**	[-21.25, -1.87]
$\gamma_{\mu x3}$	-4.800	[-16.18, 8.27]
$\gamma_{\mu x4}$	-2.638	[-16.76, 7.19]
$\gamma_{\mu x5}$	5.856***	[2.01, 10.13]
$\gamma_{\mu x6}$	13.136***	[6.12, 17.72]
$\gamma_{\mu x7}$	-1.631	[-22.51, 1.31]
$\gamma_{\mu x8}$	4.383	[-15.05, 19.00]

(g) μ^k_{xij}: $\mathbf{x}(\cdot)$

name	est.	95% CI
$\gamma_{\beta 0}$	-0.458*	[-22.67, 0.15]
$\gamma_{\beta 1}$	-0.046	[-7.68, 0.11]
$\gamma_{\beta 2}$	0.009	[-0.01, 0.18]

(h) β^k_O: $\mathbf{b}(\cdot)$

name	est.	95% CI
$\gamma_{\delta 0}$	6.061	[-3.27, 11.47]
$\gamma_{\delta 1}$	-5.236***	[-14.07, -2.58]
$\gamma_{\delta 2}$	1.771**	[0.54, 3.58]
$\gamma_{\delta 3}$	2.388	[-8.56, 15.42]
$\gamma_{\delta 4}$	-7.893*	[-22.68, 0.67]
$\gamma_{\delta 5}$	-14.245***	[-20.17, -8.20]
$\gamma_{\delta 6}$	13.017***	[6.91, 21.31]
$\gamma_{\delta 7}$	-0.050	[-2.25, 3.05]
$\gamma_{\delta 8}$	3.472***	[1.48, 7.32]
$\gamma_{\delta 9}$	0.014	[-0.28, 0.11]

(i) δ^k_{ij}: $\mathbf{d}(\cdot)$

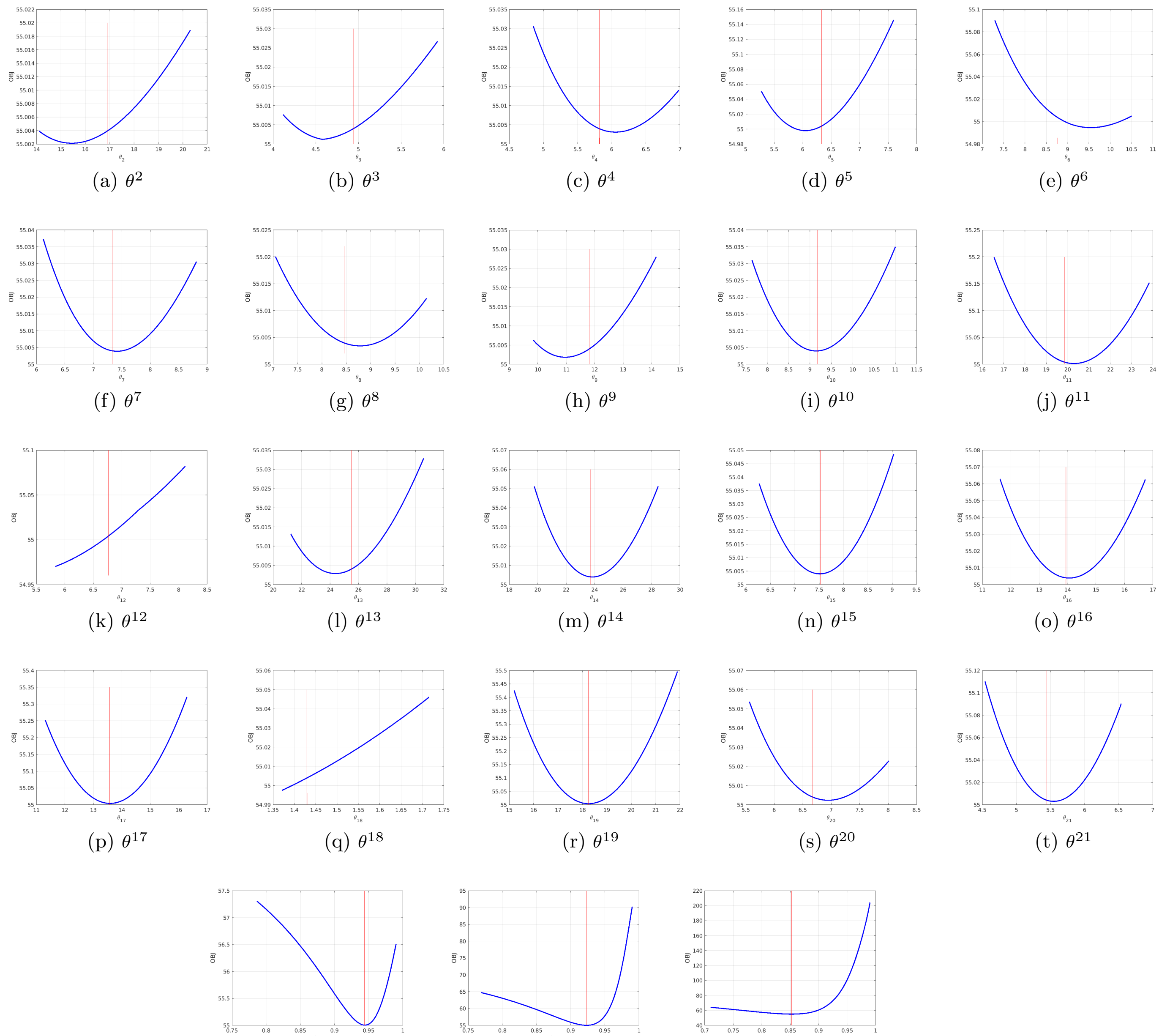
Table B.4: Full list of estimated parameters

Notes: The table lists the estimated parameters from the weighted-NLLS. ***, **, and * denote significance at the 1%, 5%, and 10% levels respectively, based on 99%, 95%, and 90% percentile confidence intervals from 200 bootstrap samples; the 95% confidence intervals are reported.

Panels (a)-(c) report the θ^k dispersion parameters; recall that θ^1 is normalized to 4. Panel (d) reports the nested-Fréchet correlation parameters, $\lambda_{(1)}$, $\lambda_{(2)}$, and $\lambda_{(3)}$, for the lower-middle, upper-middle and high income exporting countries respectively. Panel (e) reports the polynomial coefficient estimates for the $\mathbf{a}(\cdot)$ function for α^k. Panels (f)-(i) do likewise for the $\mathbf{h}(\cdot)$, $\mathbf{x}(\cdot)$, $\mathbf{b}(\cdot)$, and $\mathbf{d}(\cdot)$ functions that parameterize μ^k_{hij}, μ^k_{xij}, β^k_{ijO} and δ^k_{ij} respectively. (We refer to β^k_{ijO} interchangeably as β^k_O, since the empirical specification we adopt for this parameter only involves industry characteristics.)

Separately, the value of the weighted sum of squared errors objective function at the end of the optimization ('Fval') is 55.004. Dividing this by the sum of the observation weights and then taking the square root, the weighted root mean squared error is 0.169.

Numerical identification: Figures B.1 and B.2 illustrate how the weighted-NLLS objective function in (36) varies around the baseline parameter estimates. (For each element of Θ in turn, we plot this while holding all other elements at their baseline point estimate values.) The figures provide reassurance that the optimization routine is not caught in a region of the parameter space that is flat (in which it would be hard to search for a minima). The panels show that the objective function features a local minimum around the solution point along each dimension of Θ. The only exceptions are in Figure B.1, where the first-order conditions of θ^{12} and θ^{18} are non-binding; these point estimates are corner solutions from the restriction $\frac{(1-\alpha^k)\rho^k}{\theta^k(1-\rho^k)} < 1$ (which ensures that the dispersion of the nested-Fréchet parameters is not too large).



(a) θ^2 (b) θ^3 (c) θ^4 (d) θ^5 (e) θ^6

(f) θ^7 (g) θ^8 (h) θ^9 (i) θ^{10} (j) θ^{11}

(k) θ^{12} (l) θ^{13} (m) θ^{14} (n) θ^{15} (o) θ^{16}

(p) θ^{17} (q) θ^{18} (r) θ^{19} (s) θ^{20} (t) θ^{21}

(u) $\lambda_{(1)}$ (v) $\lambda_{(2)}$ (w) $\lambda_{(3)}$

Figure B.1: Moment Conditions, θ^k's and λ_i's

Notes: Each panel illustrates the behavior of the weighted-NLLS objective function around the baseline point estimate of the respective θ^k or λ_i parameter; the vertical line indicates the baseline point estimate.

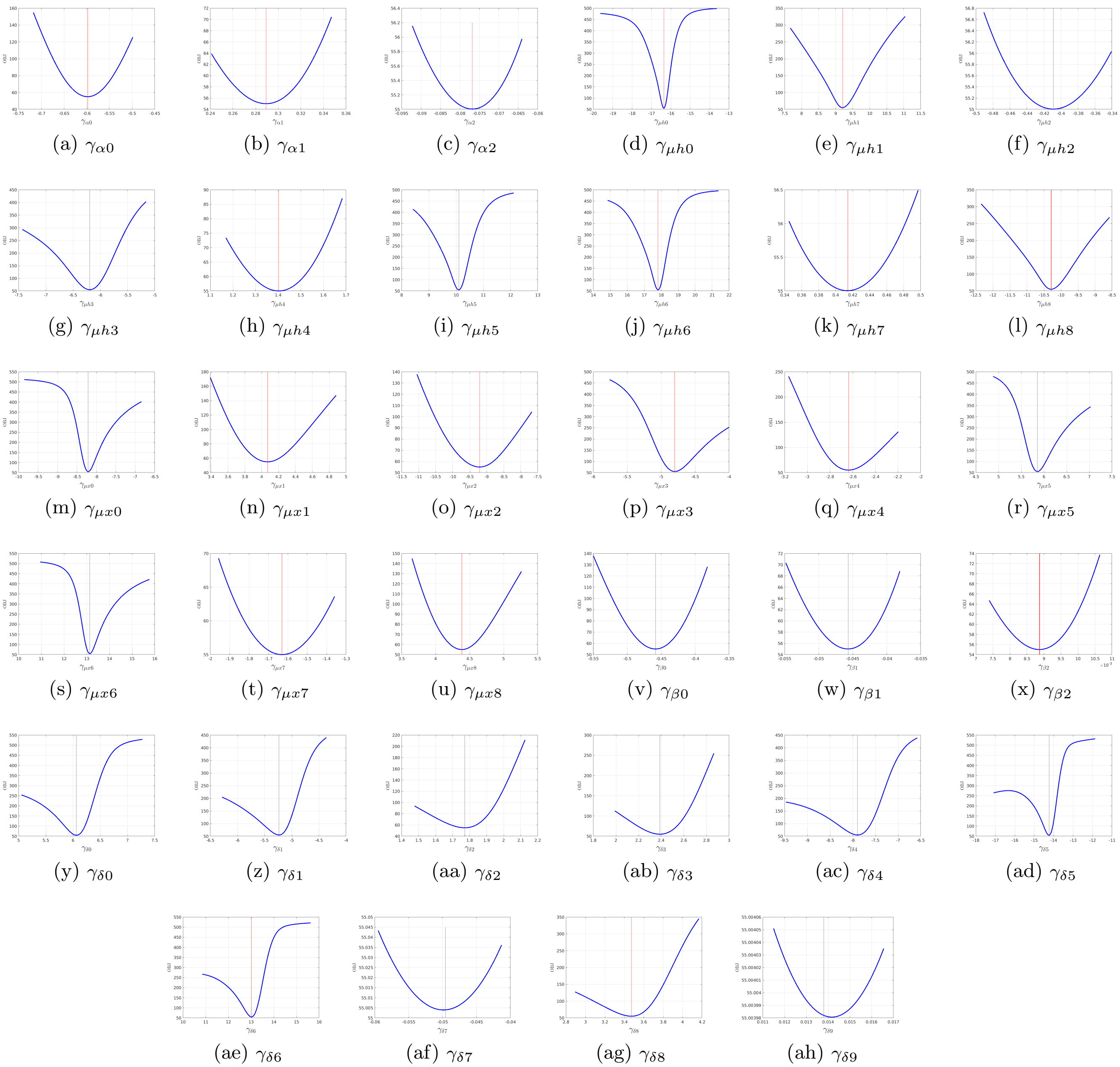
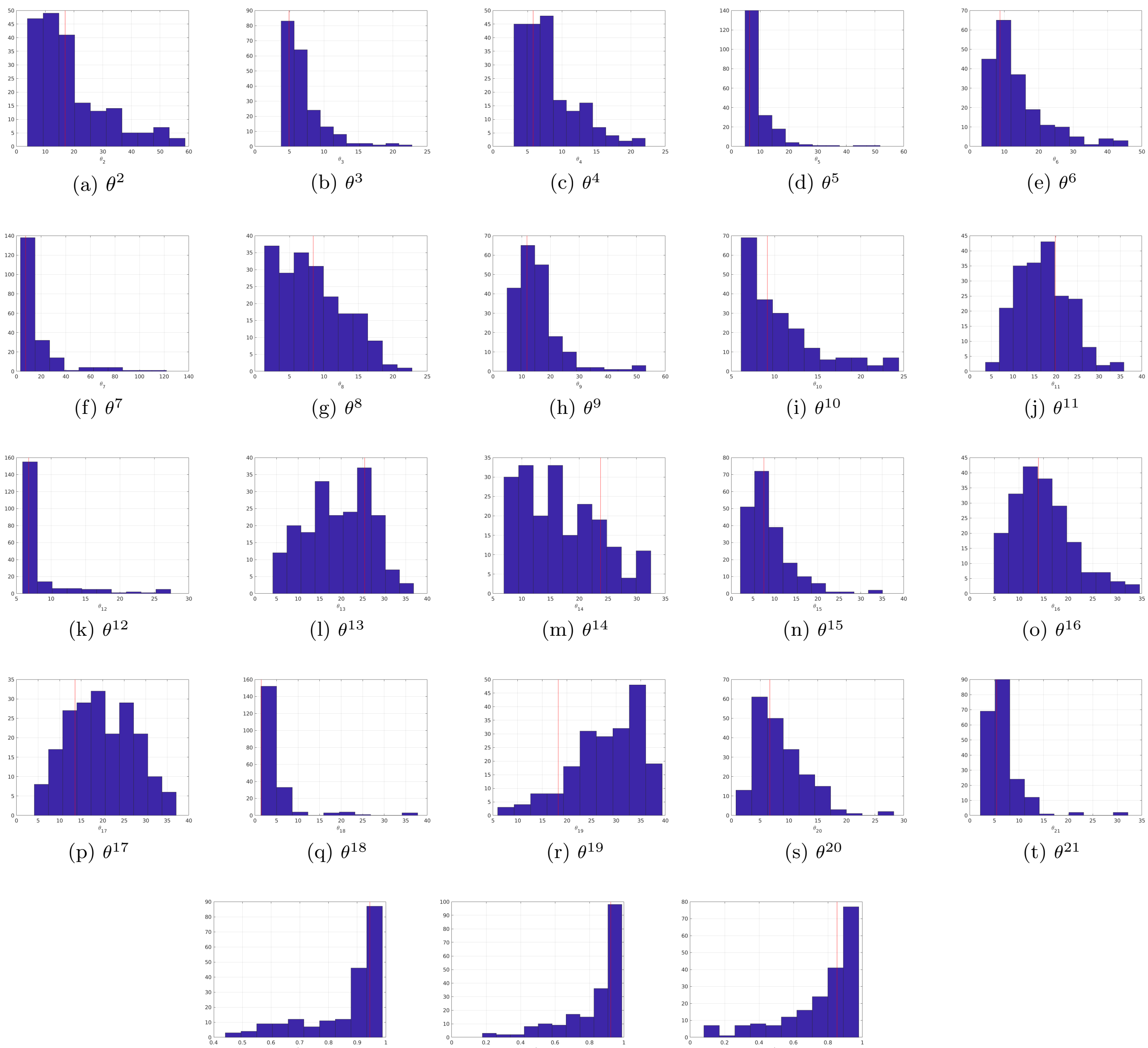


Figure B.2: Moment Conditions, $\gamma_{(\cdot)}$ coefficients in parameterizations of α^k, μ^k_{hij}, μ^k_{xij}, β^k_O, δ^k_{ij}

Notes: Each panel illustrates the behavior of the weighted-NLLS objective function around the baseline point estimate of the respective $\gamma_{(\cdot)}$ polynomial coefficients; the vertical line indicates the baseline point estimate.

Bootstrapped confidence intervals: We compute the confidence intervals via bootstrapping, based on 200 samples (resampling with replacement). In each bootstrap iteration, we apply the same PSO/LM hybrid minimization routine as described in Section B.5. Upon completion of the estimation for each of the 200 bootstrap samples, we then compute the percentile confidence interval; for example, the 95% confidence interval is defined as the interval between the 2.5th to 97.5th percentile values of the parameter across the 200 bootstrap samples. Studentized confidence intervals are not suitable for our application, as many of the bootstrapped distributions are skewed. See Figures B.3 and B.4 below for the histograms of the bootstrapped distributions for each element of Θ.



(a) θ^2 (b) θ^3 (c) θ^4 (d) θ^5 (e) θ^6

(f) θ^7 (g) θ^8 (h) θ^9 (i) θ^{10} (j) θ^{11}

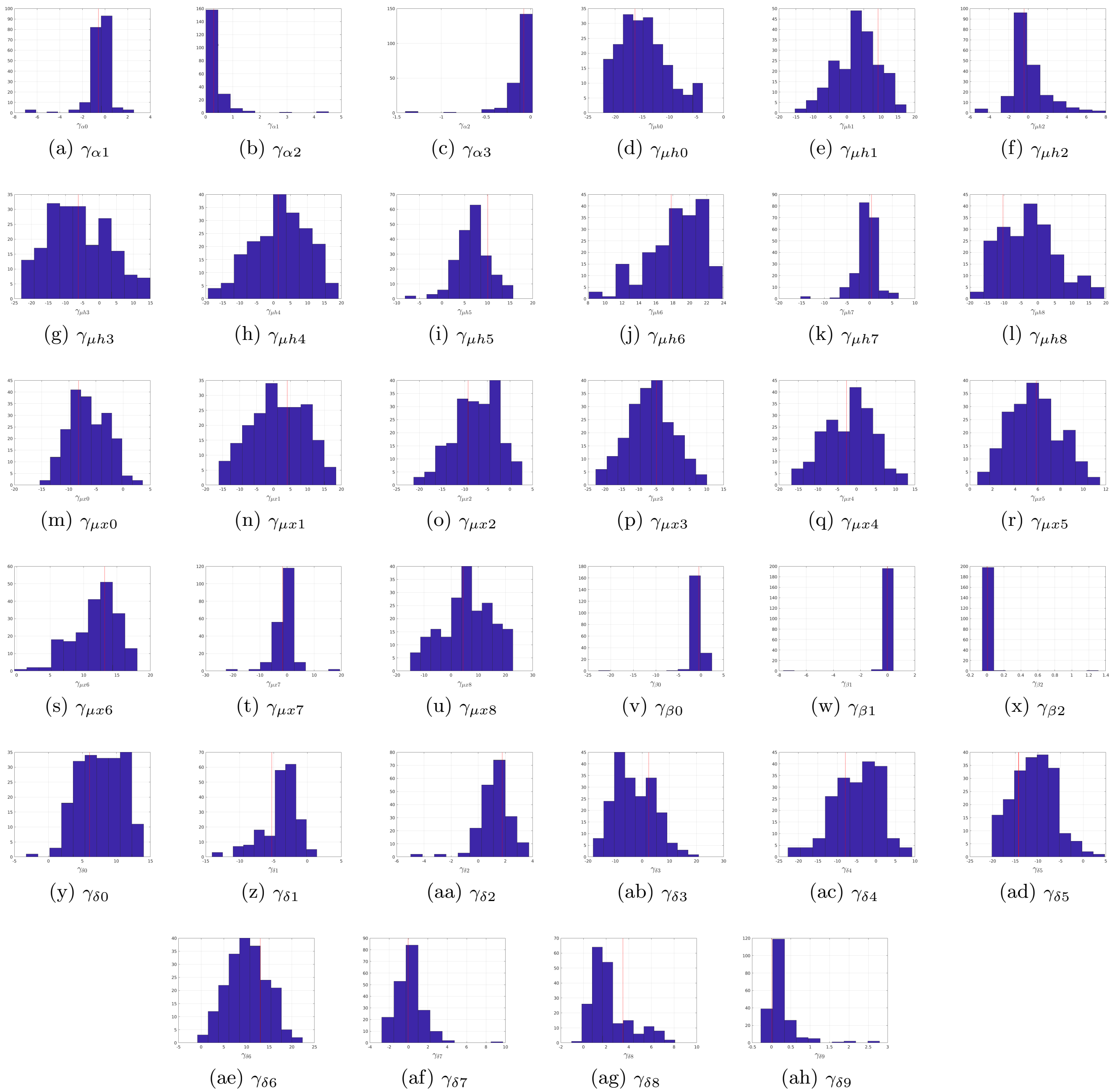
(k) θ^{12} (l) θ^{13} (m) θ^{14} (n) θ^{15} (o) θ^{16}

(p) θ^{17} (q) θ^{18} (r) θ^{19} (s) θ^{20} (t) θ^{21}

(u) $\lambda_{(1)}$ (v) $\lambda_{(2)}$ (w) $\lambda_{(3)}$

Figure B.3: Bootstrapped Distributions, θ^k's and λ_i's

Notes: The figures illustrate histograms of the bootstrapped distributions of the θ^k and $\lambda_{(\cdot)}$ parameters. The vertical line indicates the baseline point estimate.



(a) $\gamma_{\alpha 1}$ (b) $\gamma_{\alpha 2}$ (c) $\gamma_{\alpha 3}$ (d) $\gamma_{\mu h0}$ (e) $\gamma_{\mu h1}$ (f) $\gamma_{\mu h2}$

(g) $\gamma_{\mu h3}$ (h) $\gamma_{\mu h4}$ (i) $\gamma_{\mu h5}$ (j) $\gamma_{\mu h6}$ (k) $\gamma_{\mu h7}$ (l) $\gamma_{\mu h8}$

(m) $\gamma_{\mu x0}$ (n) $\gamma_{\mu x1}$ (o) $\gamma_{\mu x2}$ (p) $\gamma_{\mu x3}$ (q) $\gamma_{\mu x4}$ (r) $\gamma_{\mu x5}$

(s) $\gamma_{\mu x6}$ (t) $\gamma_{\mu x7}$ (u) $\gamma_{\mu x8}$ (v) $\gamma_{\beta 0}$ (w) $\gamma_{\beta 1}$ (x) $\gamma_{\beta 2}$

(y) $\gamma_{\delta 0}$ (z) $\gamma_{\delta 1}$ (aa) $\gamma_{\delta 2}$ (ab) $\gamma_{\delta 3}$ (ac) $\gamma_{\delta 4}$ (ad) $\gamma_{\delta 5}$

(ae) $\gamma_{\delta 6}$ (af) $\gamma_{\delta 7}$ (ag) $\gamma_{\delta 8}$ (ah) $\gamma_{\delta 9}$

Figure B.4: Bootstrapped Distributions, $\gamma_{(\cdot)}$ coefficients in parameterizations of α^k, μ^k_{hij}, μ^k_{xij}, β^k_O, δ^k_{ij}

Notes: The figures illustrate histograms of the bootstrapped distributions of the $\gamma_{(\cdot)}$ polynomial coefficients. The vertical line indicates the baseline point estimate.

Reduced-form regressions: We present further evidence that the parameter estimates we obtain (c.f., Table B.4) yield a good fit to the intrafirm trade share data. We do so by demonstrating that the intrafirm trade shares predicted by our parameter estimates correlate with country and industry observables in a manner comparable to the actual intrafirm trade share data. For this purpose, we run reduced-form regressions of the actual intrafirm trade shares (respectively, the predicted intrafirm trade shares) on the full set $\mathbf{X}^k_{ij}$ of country and industry variables (including squared and interaction terms) that we use in our empirical specifications of the $\mathbf{a}(\cdot)$, $\mathbf{h}(\cdot)$, $\mathbf{x}(\cdot)$, $\mathbf{b}(\cdot)$, and $\mathbf{d}(\cdot)$ polynomials functions that parameterize α^k, μ^k_{hij}, μ^k_{xij}, β^k_{ijO} and δ^k_{ij} respectively.

Table B.5 presents the results based on OLS regressions, run using total trade flows in the country-pair-by-industry cell as weights. Columns 1-3 are based on the observed intrafirm trade shares from the data, while Columns 4-6 use the predicted shares from our model-based parameter estimates. In addition to the OLS coefficients, we also report the Average Marginal Effect (AME) and the Inter-Quartile Effect (IQE) of each country or industry characteristic, as the regression coefficients in Columns 1 and 4 can be tricky to interpret in the presence of second-order polynomial terms. The AME of a given country or industry variable $X \in \mathbf{X}_{ij}^k$ is defined as the partial derivative of the dependent variable with respect to X (taking into account squared and interaction terms in which X may appear), evaluated at the sample mean of X; all other covariates in $\mathbf{X}_{ij}^k$ are also set at their sample mean values. The IQE is the implied difference (based on the regression coefficients) in the intrafirm trade share when we move a variable X from its 25th to the 75th percentile in the sample, while keeping all the other variables at their in-sample mean values. For interaction terms between two variables X and Y, the IQE reported is the implied change in the outcome variable if one were to move X from its 25th to 75th percentile value, while also similarly moving Y from its 25th to 75th percentile value. The AME is reported in Columns 2 and 5, while the IQE is reported in Columns 3 and 6; the standard errors are computed using the delta method.

Table B.5 confirms that the predicted and actual intrafirm trade shares line up well in terms of the signs and magnitudes of the partial correlations they exhibit with these country and industry variables (comparing Columns 1 and 4). Several key patterns stand out. First, the capital intensity of the industry correlates positively and significantly with the intrafirm trade share; this corroborates a key empirical regularity first uncovered by Antràs (2003). Second, the intrafirm trade shares correlate positively and strongly with the rule of law in the importing country; on the other hand, we do not find a significant relationship with respect to the rule of law in the exporting country. Third, a higher HQContractibility^k is associated with a reduced propensity toward integration (see in particular the negative and significant IQE in Column 3), while a higher SSContractibility^k is associated with a greater intrafirm trade share. This is in line with the predictions of the theory (c.f., Lemma 2). (Recall that a higher degree of contractibility of headquarter-provided inputs would reduce the need to incentivize the firm headquarters to engage in investments on noncontractible tasks; instead, incentivizing suppliers becomes relatively more important, hence raising the prevalence of outsourcing. The converse logic applies if instead supplier inputs are more inherently contractible.) Lastly, while several interaction terms yield statistically significant coefficients, the most consequential of these in terms of magnitudes (specifically, the IQE) is that between ROL$_j$ and Specificity^k. This is consistent with the distinct shape of the surface plots in Panels (e)-(f) of Figure 3 in the main paper, where the interaction between industry specificity and destination-country rule of law is important for explaining the share of ex-post surplus δ_{ij}^k the firm can assure itself if it were to integrate its supplier.

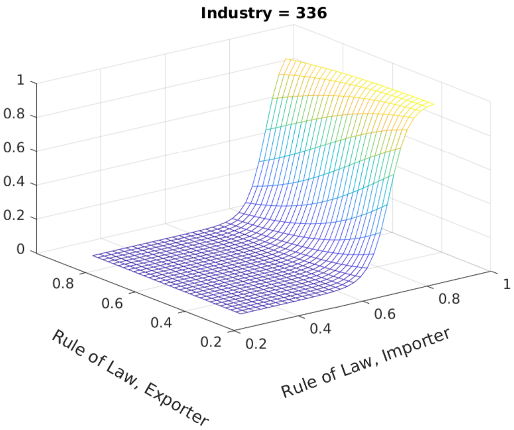
Additional surface plots: Figure B.5 presents additional surface plots to illustrate how μ_{hij}^k, μ_{xij}^k, and δ_{ij}^k behave with respect to the country and industry observables that they have been mapped to; these are based on the estimated polynomial coefficients of $\mathbf{h}(\cdot)$, $\mathbf{x}(\cdot)$, and $\mathbf{d}(\cdot)$ reported in Table B.4. In each of these surface plots, the variable ROL$_j$ is increasing along the axis that points in the northeast direction, while ROL$_i$ is increasing along the northwest axis. Panels (a) and (b) show how μ_{hij}^k as implied by the estimated $\mathbf{h}(\cdot)$ function varies with respect to rule of law in the source and destination countries; this is illustrated at the 10th percentile value of HQContractibility^k in Panel (a) on the left, and at the 90th percentile value of this industry variable in Panel (b) on the right. Panels (c)-(d) (respectively, Panels (e)-(f)) do likewise for μ_{xij}^k (δ_{ij}^k), where SSContractibility^k (Specificity^k) is the associated industry variable.

From Panels (a)-(d), we see once again that μ_{hij}^k and μ_{xij}^k are increasing in the rule of law in the destination country. However, the relationship with rule of law in the source country is much flatter; increases in ROL$_i$ are not associated with clear shifts in μ_{hij}^k or μ_{xij}^k (for any given value of ROL$_j$). There are some subtle patterns if one looks closely: For example, the μ_{xij}^k surface starts to rise at lower values of importer ROL$_j$ when the exporting country has a low ROL$_i$ (see Panels (c)-(d)). But more broadly speaking, ROL$_i$ does not appear to account for a significant slice of the variation in the implied μ_{hij}^k and μ_{xij}^k model parameters. Panels (e)-(f) indicate that δ_{ij}^k is generally increasing in ROL$_j$, but its relationship with ROL$_i$ can exhibit non-linearities.

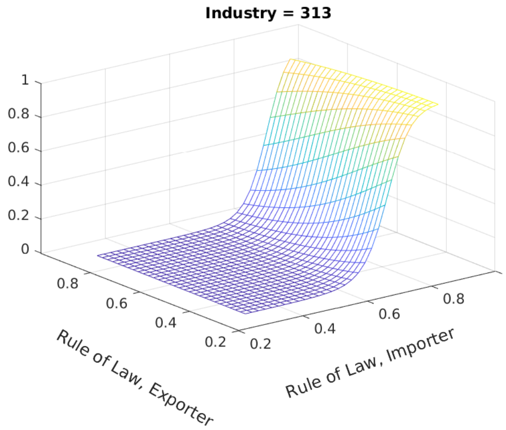
LHS = Intrafirm Trade Share	Data			Prediction		
	(1)	(2)	(3)	(4)	(5)	(6)
	coefs.	margins	f(75%)-f(25%)	coefs.	margins	f(75%)-f(25%)
ROL, imp.	2.377*	0.317**	0.062***	1.747	0.451***	0.078***
	(1.237)	(0.125)	(0.016)	(1.290)	(0.124)	(0.009)
ROL, exp.	-0.013	0.099	0.013	0.189	0.075	0.009
	(1.127)	(0.107)	(0.010)	(0.931)	(0.088)	(0.006)
Capital Intensity	0.186**	0.172**	0.218***	0.186***	0.172***	0.218***
	(0.075)	(0.077)	(0.031)	(0.063)	(0.066)	(0.018)
Specificity	2.279*	-0.306	-0.056	2.086*	-0.280	-0.049**
	(1.374)	(0.329)	(0.041)	(1.218)	(0.301)	(0.024)
Contractibility, hq	0.325	-0.367	-0.139***	-0.299	-0.410**	-0.140***
	(0.846)	(0.232)	(0.023)	(0.738)	(0.196)	(0.013)
Contractibility, ss	5.426***	1.001*	0.077***	6.315***	0.983**	0.077***
	(2.045)	(0.562)	(0.012)	(1.981)	(0.492)	(0.007)
Mean Ind.markup	-0.067	0.028	0.006	-0.044	0.023	0.010
	(0.099)	(0.024)	(0.026)	(0.094)	(0.018)	(0.015)
ROL, imp. × ROL, imp.	-0.265			0.180		
	(0.544)			(0.631)		
ROL, exp. × ROL, exp.	0.038			-0.021		
	(0.565)			(0.488)		
Capital Intensity × Capital Intensity	-0.031			-0.031		
	(0.029)			(0.024)		
Specificity × Specificity	-0.919			-0.859		
	(0.880)			(0.811)		
Contractibility, hq × Contractibility, hq	-0.567			-0.446		
	(0.605)			(0.504)		
Contractibility, ss × Contractibility, ss	-5.883**			-5.521**		
	(2.508)			(2.288)		
Mean Ind.markup × Mean Ind.markup	0.013			0.009		
	(0.013)			(0.012)		
ROL, exp. × Specificity	-0.297		-0.022**	-0.397*		-0.029***
	(0.294)		(0.011)	(0.237)		(0.006)
ROL, imp. × Specificity	-1.156***		-0.125***	-0.906***		-0.098***
	(0.351)		(0.022)	(0.325)		(0.013)
ROL, exp. × Contractibility, hq	1.288**		0.039***	1.252**		0.038***
	(0.648)		(0.007)	(0.520)		(0.004)
ROL, imp. × Contractibility, hq	-1.421*		-0.063***	-0.769		-0.034***
	(0.758)		(0.015)	(0.710)		(0.008)
ROL, exp. × Contractibility, ss	-0.909		-0.013**	-1.058**		-0.015***
	(0.693)		(0.006)	(0.527)		(0.003)
ROL, imp. × Contractibility, ss	-0.257		-0.005	-1.601**		-0.032***
	(0.701)		(0.010)	(0.761)		(0.006)
Constant	-2.188*			-2.024*		
	(1.252)			(1.087)		
N	1,926	1,926	1,926	1,926	1,926	1,926
R-squared	0.384		-	0.653		-
Fixed Effects	None	N/A	N/A	None	N/A	N/A

Table B.5: Reduced-Form Relationships between the Intrafirm Trade Share and Observables, OLS

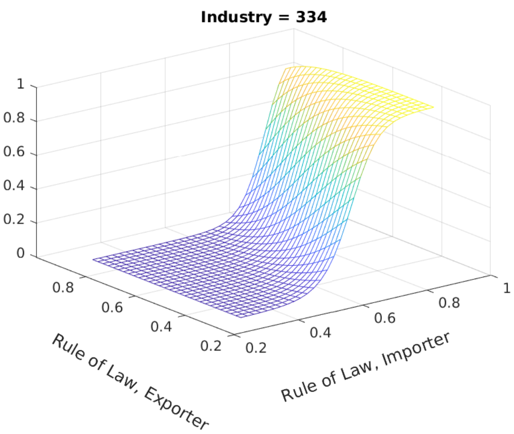
Notes: This table reports the OLS results from regressing the intrafirm trade share on country and industry variables, using total trade flows in the country-pair-by-industry cell as weights. Standard errors are two-way clustered at the exporter-industry and the importer-industry levels. ***, **, and * denote significance at the 1%, 5%, and 10% levels respectively. Columns 1-3 use the *observed* intrafirm trade shares from the data as the dependent variable, while Columns 4-6 use the *predicted* intrafirm trade shares from our estimated model. Columns 2 and 5 report average marginal effects for each country and industry variable, while Columns 3 and 6 report the inter-quartile effects; standard errors in these columns are computed using the delta method based on the variance-covariance matrices from Columns 1 and 4 respectively.



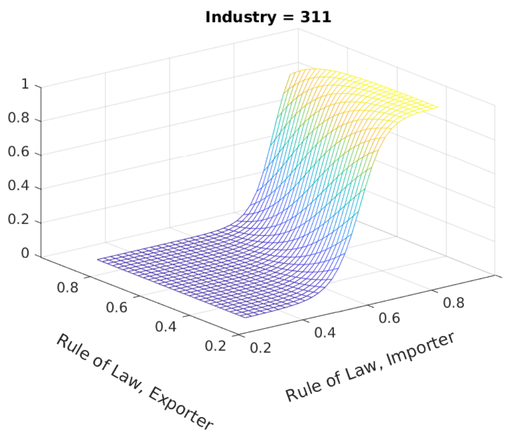
(a) μ^k_{hij}, Low HQ Contractibility



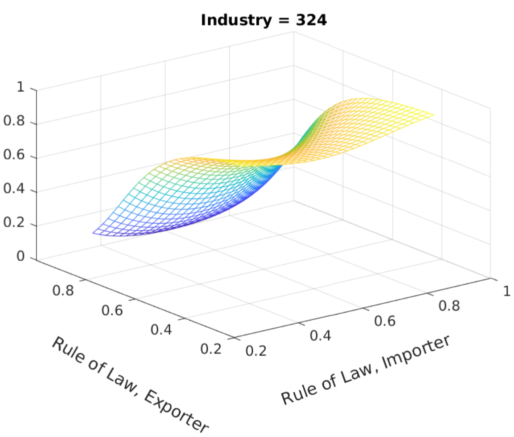
(b) μ^k_{hij}, High HQ Contractibility



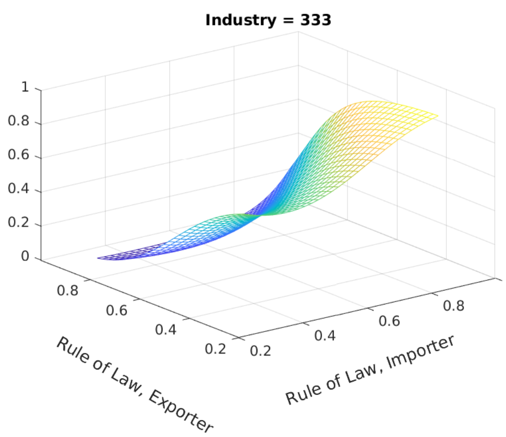
(c) μ^k_{xij}, Low Supplier Contractibility



(d) μ^k_{xij}, High Supplier Contractibility



(e) δ^k_{ij}, Low Specificity



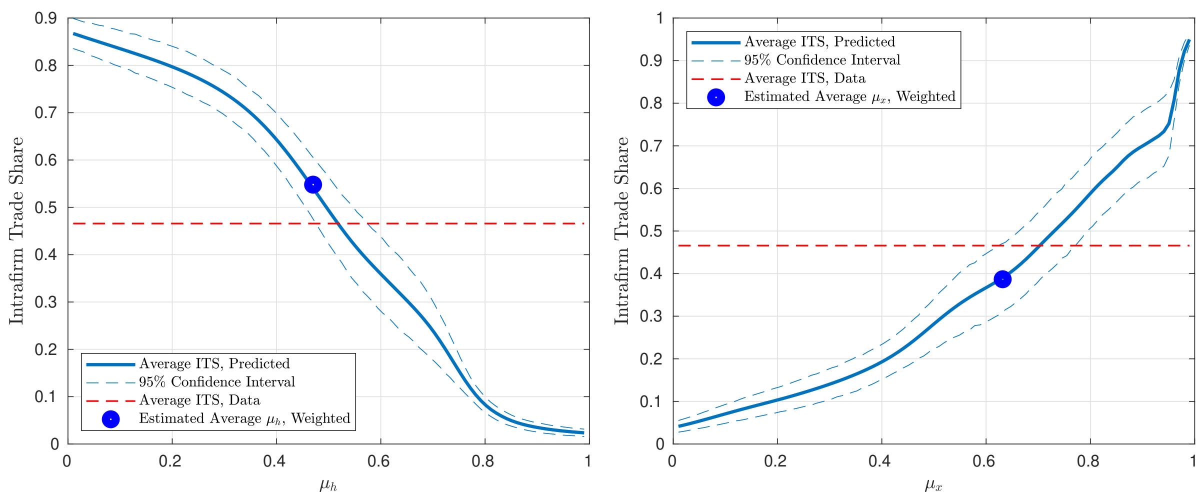
(f) δ^k_{ij}, High Specificity

Figure B.5: Additional Surface Plots: μ^k_{hij}, μ^k_{xij}, and δ^k_{ij}

Notes: The figures present the surface plots of the μ^k_{xij}, μ^k_{hij}, and δ^k_{ij} functions. The plots are based on the estimated $\gamma_{(\cdot)}$ coefficients of the $\mathbf{h}(\cdot)$, $\mathbf{x}(\cdot)$, and $\mathbf{d}(\cdot)$ functions. The northeast axes plot ROL_j, while the northwest axes plot ROL_i; the left column sets the industry characteristic at its 10th percentile value, while the right column sets this characteristic at its 90th percentile value. The associated industry characteristics are $\text{HQContractibility}^k$ for the μ^k_{hij} surface, $\text{SSContractibility}^k$ for μ^k_{xij}, and Specificity^k for δ^k_{ij}.

Pinning down the average levels of μ^k_{hij} and μ^k_{xij}: We furnish intuition on how the average intrafirm trade shares implied by the model vary in relation to the average levels of the contractibility parameters, μ^k_{hij} and μ^k_{xij}. This in turn highlights how the average levels of μ^k_{hij} and μ^k_{xij} are pinned down in our structural estimation to be in line with the average intrafirm trade shares in the actual data. (Note that all averages here are trade-weighted, using the trade value $\tilde{t}^k_{ijV} + \tilde{t}^k_{ijO}$ as weights.)

Consider first how the average intrafirm trade share varies with μ^k_{hij}. To highlight this, we set μ^k_{hij} to a uniform value across all observations in our sample and vary this common μ^k_{hij}, while keeping the other model parameters (including the μ^k_{xij}'s) at their values from the baseline weighted-NLLS estimation. Panel (a) in Figure B.6 plots the predicted average intrafirm trade shares across all observations against the level of μ^k_{hij}; the 95% confidence intervals illustrated by the dotted blue lines are based on 2,000 bootstrap samples (drawn with replacement). As one would expect from part (ii) of Lemma 2, a higher headquarter task contractibility lowers the average intrafirm trade share (*ceteris paribus*), since it progressively becomes more important to incentivize supplier effort through outsourcing. The average intrafirm trade share of 0.47 in the actual data is highlighted by the red dotted horizontal line. Note that the average level of μ^k_{hij} implied by our weighted-NLLS estimates (coincidentally also 0.47, illustrated by the blue circle) lines up well with this average intrafirm trade share, as the 95% confidence interval that vertically spans the blue circle covers the average intrafirm trade share value of 0.47. Panel (b) in Figure B.6 performs the analogous exercise with a common value of μ^k_{xij}, conditioning on the estimated values of all other parameters (including μ^k_{hij}). The predicted average intrafirm trade share is increasing with respect to μ^k_{xij}, as it instead becomes more important to incentivize effort from the firm headquarters via integration when supplier inputs become more contractible (c.f., Lemma 2). The average μ^k_{xij} value of 0.63 implied by our estimates (blue circle) is once again consistent with the average intrafirm trade share seen in the data (red dotted line), as the 95% confidence interval covers the actual intrafirm trade share of 0.47.



(a) μ^k_{hij} (b) μ^k_{xij}

Figure B.6: Pinning Down the Average Levels of μ^k_{hij} and μ^k_{xij}

Notes: Panel (a) plots the predicted average intrafirm trade share against the common value of μ^k_{hij} assumed across all observations (holding all other parameters including the μ^k_{xij}'s at their estimated values). Panel (b) performs the analogous exercise, with a common value of μ^k_{xij} assumed across all observations that is then varied along the horizontal axis. The blue dashed lines are the 95% confidence interval bounds of the predicted average intrafirm trade share, based on 2,000 bootstraps (sampling with replacement). The red dashed line indicates the average intrafirm trade share in the data. The blue circles are the average values of μ^k_{hij} and μ^k_{xij} implied by the weighted-NLLS estimation. All averages are trade-weighted average values.

B.7 Alternative Empirical Specifications

We report the results from two alternative empirical specifications. First, we estimate a single β_O bargaining parameter – that is common and constant across all i, j, and k – instead of assuming it to be a function of the industry-level markup. The second alternative experiments with other country-level variables. In the baseline model, we used the country-level measure of the institutional rule of law (ROL) in all three polynomials, $\mathbf{h}(\cdot)$, $\mathbf{x}(\cdot)$, and $\mathbf{d}(\cdot)$. As an alternative, we replace the rule of law index in the $\mathbf{d}(\cdot)$ function that parameterizes δ_{ij}^k with a measure of "Recovery Rates", for both source and destination countries. This is drawn from the World Bank Doing Business dataset, and is defined as the cents on the dollar that can be recovered by secured creditors from reorganization, liquidation, or debt enforcement in insolvency proceedings. As explained in the main paper, although not all sourcing contracts involve debt, this measure may nevertheless speak to the extent to which the firm can successfully recover value from the remaining proceeds should there be a breakdown in a supplier relationship. The rest of the empirical specification for $\mathbf{d}(\cdot)$ remains the same, as in equation (B.10).

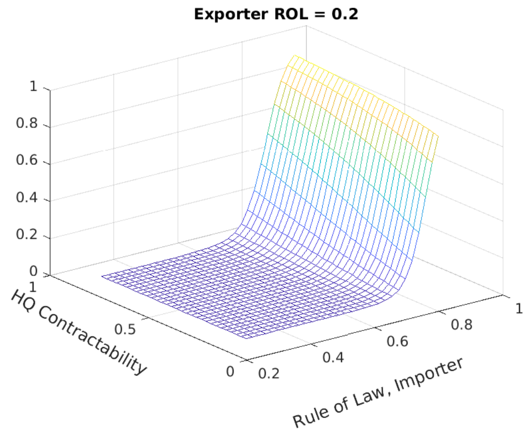
	Baseline	Single β_O	Recovery Rates in δ
F-val	55.004	55.802	52.313
Intrafirm Trade Share, Mean	0.464	0.466	0.461
Intrafirm Trade Share, Standard Deviation	2.566	2.576	2.633
Intrafirm Trade Share, Correlation with Baseline	1.000	0.976	0.688
α^k, Mean	0.349	0.450	0.447
μ_{xij}^k, Mean	0.522	0.507	0.161
μ_{hij}^k, Mean	0.426	0.409	0.164
δ_{ij}^k, Mean	0.404	0.403	0.480
β_O, Mean	0.377	0.447	0.307
$\mu_x = \mu_h = 1$, Mean Welfare Change	0.093	0.112	0.167

Table B.6: Alternative Estimation Specifications

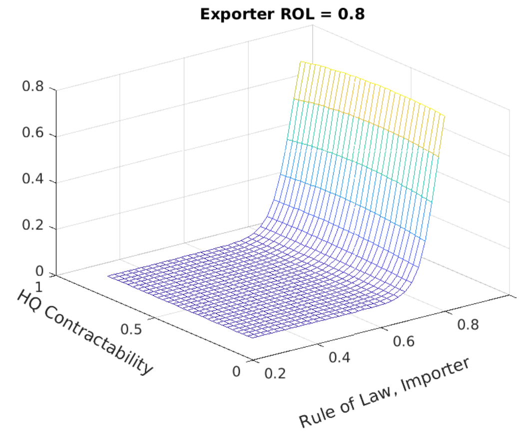
Notes: The first column reports the key features of the quantified model for our baseline specification. The second column estimates a single β_O bargaining parameter across all observations, while the third column uses the "Recovery Rates" measure in lieu of ROL in the δ_{ij}^k. All averages in the first and second panels are trade-weighted averages.

Table B.6 compares the results from these alternative specifications (second and third columns) to the baseline (first column). The residual sum of squares ('F-val') and the predicted intrafirm trade shares from the alternative specifications are very similar to those from the baseline, as reported in the top panel. Moreover, the predicted intrafirm trade shares are also highly correlated with those from the baseline, although this is slightly weaker in the "Recovery Rate in δ" specification. The second panel of the table shows that the implied values for the parameters – α^k, μ_{hij}^k, μ_{xij}^k, δ_{ij}^k, and β_{ijO}^k – from the alternative specifications are also broadly consistent with the baseline case, in terms of their weighted-mean values. Note though that the average μ_{hij}^k and μ_{xij}^k in the "Recovery Rate in δ" specification are lower than those in the baseline. The surface plots in Figure B.7 show that ROL_j in the importing country is once again the main driver of variation in μ_{hij}^k and μ_{xij}^k (Panels (a)-(d)). On the other hand, δ_{ij}^k does not appear to be shaped in a distinct way by Specificity^k and the "Recovery Rate" in the importing country (Panels (e)-(f)); the implied δ_{ij}^k values also exhibit less variation – sitting between 0.5 and 0.6 – when compared to our baseline case (compared to Panels (e)-(f) in Figure 3).

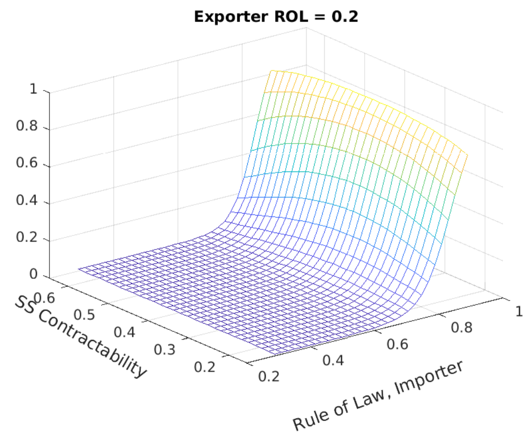
In the last row of Table B.6, we repeat the main counterfactual exercise of removing contracting frictions in global sourcing by setting $\mu_{xij}^k = \mu_{hij}^k = 1$ for all i, j, and k. The average welfare gain of 9.2% implied by our baseline estimates is smaller relative to that seen under the two alternative specifications (11.2% and 16.7%, respectively). The latter welfare gains are larger because of the lower average μ_{hij}^k and μ_{xij}^k obtained under these alternative specifications. We have adopted the baseline case in Column 1 as our preferred specification due to the more conservative welfare gain numbers this yields, as well as the fact that those baseline estimates yield a more intuitive mapping between δ_{ij}^k and its associated country and industry variables (ROL and Specificity^k).



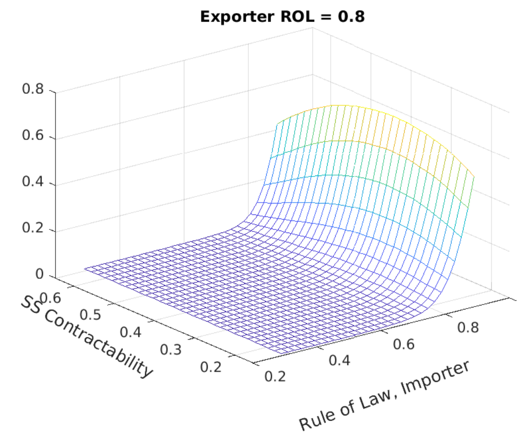
(a) μ^k_{hij}, Low Exporter ROL



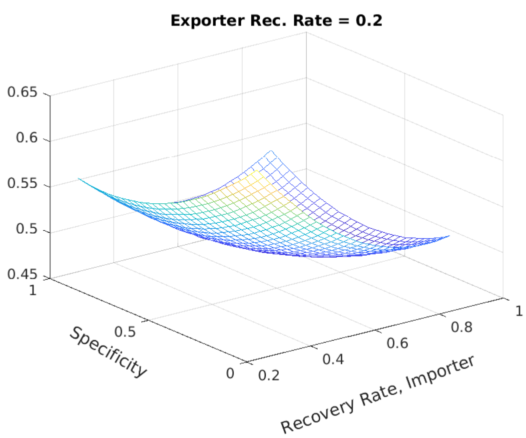
(b) μ^k_{hij}, High Exporter ROL



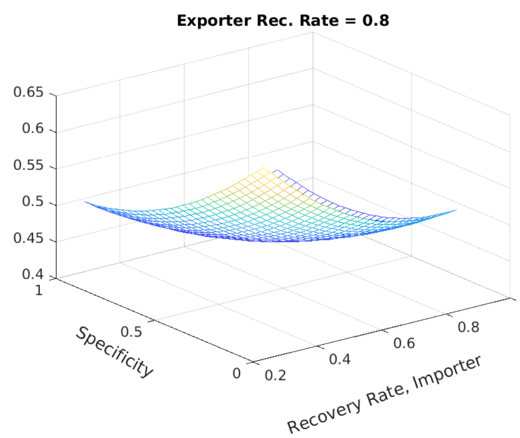
(c) μ^k_{xij}, Low Exporter ROL



(d) μ^k_{xij}, High Exporter ROL



(e) δ^k_{ij}, Low Exporter Recovery Rate



(f) δ^k_{ij}, High Exporter Recovery Rate

Figure B.7: Estimated Surfaces: μ^k_{hij}, μ^k_{xij}, and δ^k_{ij}

Notes: The figures present the surface plots of the μ^k_{hij}, μ^k_{xij}, and δ^k_{ij} functions. The plots are based on the estimated $\gamma_{(\cdot)}$ coefficients of the $\mathbf{h}(\cdot)$, $\mathbf{x}(\cdot)$, and $\mathbf{d}(\cdot)$ functions, from the alternative specification in which we use the "Recovery Rates" measure in lieu of ROL in the δ^k_{ij} function. The northeast axes plot the importing country variable, while the northwest axes plot the associated industry characteristic; the left column sets the exporting country variable at its 10th percentile value, while the right column sets the exporting country variable at its 90th percentile value. The associated industry characteristics are HQContractibility^k for the μ^k_{hij} surface, SSContractibility^k for μ^k_{xij}, and Specificity^k for δ^k_{ij}.

B.8 Sensitivity Analysis

In this appendix section, we study the sensitivity of the parameter estimates obtained in our baseline specification to perturbations in the moment conditions in the weighted-NLLS, following the methods advanced in Andrews et al. (2017). We first describe how the sensitivity measures are computed in the context of weighted-NLLS.

We write down the data generating process in a general form as follows:

$$y_i = f(\mathbf{x}_i, \Theta) + u_i, \quad i = 1, \cdots, N,$$

where y_i is the dependent variable; $\mathbf{x}_i$ is a vector of independent variables; Θ is the parameter vector of size M; $i = 1, \cdots, N$ indexes the observations; and $f(\cdot)$ is a non-linear function. The identifying assumption is $\mathbb{E}[u|\mathbf{x}] = 0$. In our context, y_i corresponds to the observed intrafirm trade share, the $\mathbf{x}_i$'s are the country and industry covariates (ROL_i, ROL_j, log real capital per worker, $\text{HQContractibility}^k$, $\text{SSContractibility}^k$, Specificity^k) that are collected in $\mathbf{x}$; and $f(\cdot)$ refers to the expression for the predicted intrafirm trade share by value in equation (35) in the main text. Our weighted-NLLS approach to the estimation problem solves:

$$\min_{\Theta} \sum_{i=1}^{N} w_i \left[y_i - f(\mathbf{x}_i, \Theta)\right]^2, \tag{B.11}$$

where the weight w_i associated with observation i is the total trade flow value, $\tilde{t}^k_{ijV} + \tilde{t}^k_{ijO}$.

Re-formulated as a Method of Moments estimator, the estimator in (B.11) is the solution to M moment conditions, which are the first-order conditions with respect to the elements of Θ:

$$g_m(\Theta) = \frac{1}{N} \sum_{i=1}^{N} w_i \left[y_i - f(\mathbf{x}_i, \Theta)\right] \frac{\partial f(\mathbf{x}_i, \Theta)}{\partial \Theta_m} = 0, \quad m = 1, \cdots, M.$$

We refer to $g_m(\Theta)$ as the m-th moment function. Let $G(\Theta)$ be the M-by-M Jacobian matrix of $\{g_m(\Theta)\}_{m=1}^{M}$:

$$G(\Theta) = \begin{bmatrix} \frac{\partial g_1(\Theta)}{\partial \Theta_1} & \cdots & \frac{\partial g_1(\Theta)}{\partial \Theta_M} \\ \vdots & & \vdots \\ \frac{\partial g_M(\Theta)}{\partial \Theta_1} & \cdots & \frac{\partial g_M(\Theta)}{\partial \Theta_M} \end{bmatrix}.$$

Following Andrews et al. (2017), the sensitivity matrix Λ is given by the following M-by-M matrix:

$$\Lambda = -(G'G)^{-1}G' = -G^{-1}. \tag{B.12}$$

The element in the m-th row and k-th column of the Jacobian matrix G is

$$\frac{\partial g_m(\Theta)}{\partial \Theta_k} = -\frac{1}{N} \sum_{i=1}^{N} w_i \frac{\partial f(\mathbf{x}_i, \Theta)}{\partial \Theta_k} \frac{\partial f(\mathbf{x}_i, \Theta)}{\partial \Theta_m} + \frac{1}{N} \sum_{i=1}^{N} w_i \left[y_i - f(\mathbf{x}_i, \Theta)\right] \frac{\partial^2 f(\mathbf{x}_i, \Theta)}{\partial \Theta_m \partial \Theta_k}. \tag{B.13}$$

Under the assumption that the weighted-NLLS is correctly specified so that $\mathbb{E}[u|\mathbf{x}] = 0$, the second term involving the Hessian matrix is asymptotically zero. To see this, denote $h_i(\mathbf{x}, \Theta) = \frac{\partial^2 f(\mathbf{x}_i, \Theta)}{\partial \Theta_m \partial \Theta_k}$; we have:

$$\begin{aligned} \frac{1}{N} \sum_{i=1}^{N} w_i \left[y_i - f(\mathbf{x}_i, \Theta)\right] \frac{\partial^2 f(\mathbf{x}_i, \Theta)}{\partial \Theta_m \partial \Theta_k} &= \frac{1}{N} \sum_{i=1}^{N} w_i u_i h_i(\mathbf{x}, \Theta) \\ &\xrightarrow{p} \mathbb{E}[w \cdot u \cdot h(\mathbf{x})] = \mathbb{E}\left[\mathbb{E}[w \cdot u \cdot h(\mathbf{x})|\mathbf{x}]\right] = \mathbb{E}\left[w h(\mathbf{x}) \times \mathbb{E}[u|\mathbf{x}]\right] = 0. \end{aligned}$$

Intuitively, the above result states that under the identifying assumption, the covariance between the error term and any function of $\mathbf{x}$ is zero. In this context, the Hessian matrix is simply a function of $\mathbf{x}$.

Relationship to Gauss-Newton regressions: Standard NLLS algorithms based on Newton's method such as Levenberg-Marqardt do not compute the G matrix directly. Instead, the algorithm computes the N-by-M Jacobian matrix of each observation with respect to the parameters, denoted as $F(\Theta)$:

$$F(\Theta) = \begin{bmatrix} \frac{\partial f_1(\Theta)}{\partial \Theta_1} & \cdots \frac{\partial f_1(\Theta)}{\partial \Theta_M} \\ \vdots & \vdots \\ \frac{\partial f_N(\Theta)}{\partial \Theta_1} & \cdots \frac{\partial f_N(\Theta)}{\partial \Theta_M} \end{bmatrix}.$$

To compute the sensitivity matrix, one can readily express Λ as a function of F. Denote the N-by-N diagonal weight matrix as W, in which the i-th diagonal element is w_i. With the simplification of the m-th moment noted in (B.13), we can express the G matrix as:

$$G = -\frac{1}{N} F'WF. \tag{B.14}$$

To see this, note that the element in the m-th row and k-th column of G is the inner product of the m-th row of F' and k-th column of F:

$$\frac{\partial g_m(\Theta)}{\partial \Theta_k} = -\frac{1}{N}\left[\frac{\partial f(\mathbf{x}_1,\Theta)}{\partial \Theta_m} \cdots \frac{\partial f(\mathbf{x}_N,\Theta)}{\partial \Theta_m}\right] \begin{bmatrix} w_1 \frac{\partial f(\mathbf{x}_1,\Theta)}{\partial \Theta_k} \\ \vdots \\ w_n \frac{\partial f(\mathbf{x}_N,\Theta)}{\partial \Theta_k} \end{bmatrix} = -\frac{1}{N}\sum_{i=1}^{N} w_i \frac{\partial f(\mathbf{x}_i,\Theta)}{\partial \Theta_k}\frac{\partial f(\mathbf{x}_i,\Theta)}{\partial \Theta_m}.$$

With equation (B.14) in mind, we can re-write the sensitivity matrix from (B.12) as:

$$\Lambda = -G^{-1} = \left(\frac{1}{N}F'WF\right)^{-1}. \tag{B.15}$$

Lastly, note that the asymptotic variance matrix of $\hat{\Theta}$ from a weighted-NLLS is:

$$\Sigma = \left(F'WF\right)^{-1} F'W\Omega WF \left(F'WF\right)^{-1}.$$

The sensitivity matrix in the context of weighted-NLLS is similar to that in the context of GMM: it is the outer sandwich of the Σ matrix.

Omitted variable bias: We can now consider omitted variable bias (OVB) in the context of weighted-NLLS. Suppose that the true data-generating process is:

$$y_i = f(\mathbf{x}_i,\Theta) + h(z_i) + u_i,$$

where u_i is the true error term that satisfies $E(u|\mathbf{x}) = 0$, and $h(z_i)$ is a non-linear function of some omitted variable z_i. Suppose that $\text{cov}(x^{(r)}, h) \neq 0$, where $x^{(r)}$ is one of the covariates in $\mathbf{x}_i$, so that the estimation of Θ could be biased if the researcher runs the weighted-NLLS assuming $y_i = f(\mathbf{x}_i,\Theta) + v_i$. In our context, we might have omitted some variables that are correlated with both the importing country's rule of law and the intrafirm trade share. As a result, the identifying assumption of the weighted-NLLS will be violated, i.e., $E(v|\mathbf{x}) \neq 0$, and thus the coefficient estimates related to ROL_j such as $\gamma_{\mu x5}, \gamma_{\mu x6}, \gamma_{\mu h5}$, $\gamma_{\mu h6}, \gamma_{\delta 5}$, and $\gamma_{\delta 6}$ (see equations (B.7), (B.8), and (B.10)) could be biased.

To gauge the magnitude of the potential bias, we derive the perturbed moment condition, $\widetilde{g}_m(\Theta)$, as follows:

$$\widetilde{g}_m(\Theta) = \frac{1}{N}\sum_{i=1}^{N} w_i \left[y_i - f(\mathbf{x}_i,\Theta) - h(z_i)\right]\frac{\partial f(\mathbf{x}_i,\Theta)}{\partial \Theta_m}, \quad m = 1,\cdots,M.$$

Note that as $h(z_i) \to 0$, we would have $\widetilde{g}_m(\Theta) \to g_m(\Theta)$, so the original moment condition is the limit case in which the omitted variable approaches zero. Following the notation in Andrews et al. (2017), we then compute the biases in each parameter as:

$$\begin{bmatrix} \Theta_1 - \tilde{\Theta}_1 \\ \Theta_2 - \tilde{\Theta}_2 \\ \vdots \\ \Theta_M - \tilde{\Theta}_M \end{bmatrix} = \Lambda \begin{bmatrix} g_1(\Theta) - \widetilde{g}_1(\Theta) \\ g_2(\Theta) - \widetilde{g}_2(\Theta) \\ \vdots \\ g_M(\Theta) - \widetilde{g}_M(\Theta) \end{bmatrix}. \tag{B.16}$$

Sensitivity of our parameter estimates to OVB: Now suppose that the omitted variable, $h(z_i)$, is a linear function of a regressor, x_i, subject to an iid shock. In particular, assume:

$$h_i = \delta x_i + \epsilon_i,$$

in which ϵ_i is a Gaussian white noise term with standard error σ_ϵ. It is straightforward to see that the standard deviation of the omitted variable and its correlation with the regressors can be computed as follows:

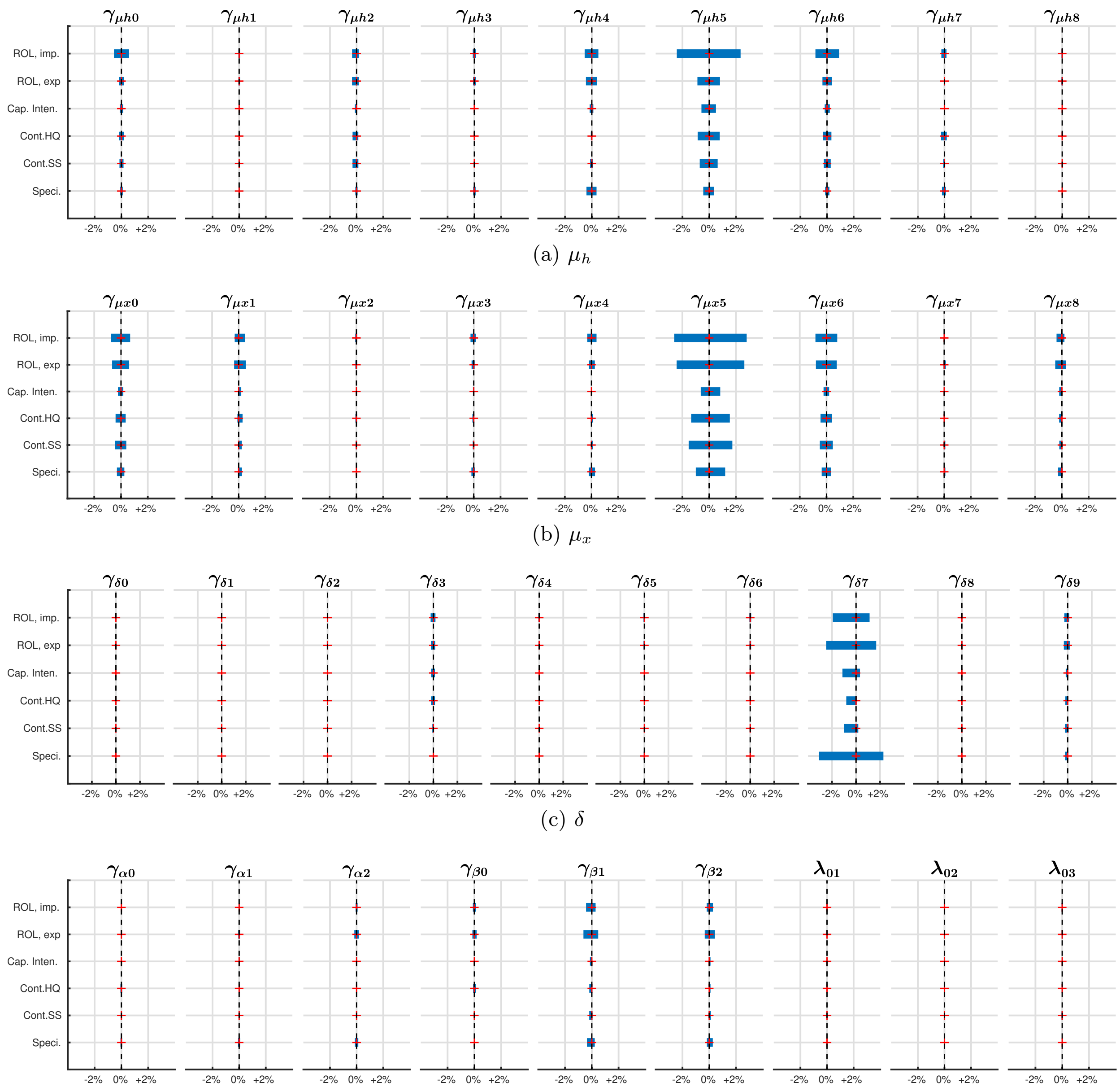
$$\sigma_h = \sqrt{\delta^2\sigma_x^2 + \sigma_\epsilon^2}, \text{ and}$$
$$\rho_{h,x} = \frac{\delta\sigma_x^2}{\sigma_h\sigma_x} = \frac{\delta\sigma_x}{\sigma_h} = \frac{\delta\sigma_x}{\sqrt{\delta^2\sigma_x^2 + \sigma_\epsilon^2}},$$

where σ_x is the standard deviation of the regressor. The above expressions offer a simple way to simulate potential omitted variables and gauge their impact on the baseline parameter estimates. Given a target standard deviation in the potential omitted variables, σ_h, and a target correlation between it and the included covariate, $\rho_{h,x}$, we can invert the above system of equations to infer the implied values of σ_ϵ and δ:

$$\sigma_\epsilon = \sigma_h\sqrt{1 - \rho_{h,x}^2}, \text{ and}$$
$$\delta = \rho_{h,x}\frac{\sigma_h}{\sigma_x}.$$

We set the target standard deviation in the potential omitted variable, σ_h, to be equal to the standard error of the residual error term in our weighted-NLLS, $\sigma_{\hat{u}}$; this maximally attributes the residual variation to the potential omitted variable. We vary the target correlation parameter $\rho_{h,x}$ between $[-0.9, 0.9]$. For each pair of values of σ_h and $\rho_{h,x}$, we use the above equations to back out the implied σ_ϵ and δ, and then use these to simulate $h_i = \delta x_i + \epsilon_i$, where the ϵ_i's are iid draws from a mean-zero Gaussian distribution with standard error σ_ϵ. We can then compute the resulting parameter bias using equation (B.16).

Figure B.8 illustrates the range of potential parameter bias (varying $\rho_{h,x}$ between ± 0.9) that we obtain through the above approach. We perform the procedure just described in turn for each of the covariates: ROL_i, ROL_j, log real capital per worker, $\text{HQContractibility}^k$, $\text{SSContractibility}^k$, and Specificity^k. Reassuringly, the potential bias from OVB is small. For example, even with a sizeable potential for OVB – with a high correlation with the regressor – the largest bias (in percentage terms) that we obtain for a parameter estimate is with $\gamma_{\mu x5}$, the coefficient of ROL_j in the $\mathbf{x}(\cdot)$ function for μ_{xij}^k. The absolute bias for this coefficient is 0.16, implying a bias relative to the point estimate of only $0.16/5.856 \approx 2.7\%$. The only other parameters where there is sensitivity to potential omitted variables of a similar magnitude are the $\gamma_{\mu h5}$ coefficient (which speaks to the effect of importer ROL on μ_{hij}^k) and the $\gamma_{\delta 7}$ coefficient (for the interaction term between specificity and importer ROL). The sensitivity of all other parameters estimated in Θ is muted in comparison.



(a) μ_h

(b) μ_x

(c) δ

(d) α, β_O, λ

Figure B.8: Sensitivity Analysis for Baseline Parameter Estimates

Notes: The sensitivity of the baseline parameter estimates to a perturbation in the moment conditions is illustrated; the perturbation simulated is related to potential omitted variables that could be correlated in turn to each of the observed covariates (arrayed vertically). The sensitivity is expressed as a percentage of the point estimate. See Appendix B.8 for details.

C Counterfactual Exercises

C.1 Hat Algebra

For any given variable X, let X' denote the change in X, and $\widehat{X} \equiv X'/X$ denote the corresponding proportional change relative to the initial value. Given counterfactual values of μ^k_{hij}, μ^k_{xij}, $\beta^k_{ij\chi}$ and/or d^k_{ij}, the system of equations to solve the model in changes is as follows:

$$(\zeta^k_{ij})' = 1 - \rho^k + \rho^k\alpha^k(\mu^k_{hij})' + \rho^k(1-\alpha^k)(\mu^k_{xij})' \tag{C.1}$$

$$(\zeta^k_{ij\chi})' = 1 - \rho^k\alpha^k\left(1-(\mu^k_{hij})'\right)(\beta^k_{ij\chi})' - \rho^k(1-\alpha^k)\left(1-(\mu^k_{xij})'\right)\left(1-(\beta^k_{ij\chi})'\right) \tag{C.2}$$

$$(B^k_{ij\chi})' = \left(\frac{(\zeta^k_{ij\chi})'}{(\zeta^k_{ij})'}\right)^{\frac{(\zeta^k_{ij})'}{\rho^k(1-\alpha^k)}} \left((\beta^k_{ij\chi})'\right)^{\frac{\alpha^k}{1-\alpha^k}\left(1-(\mu^k_{hij})'\right)} \left(1-(\beta^k_{ij\chi})'\right)^{1-(\mu^k_{xij})'} \tag{C.3}$$

$$(B^k_{ij})' = \left(\left((B^k_{ijV})'\right)^{\frac{\theta^k}{1-\lambda_i}} + \left((B^k_{ijO})'\right)^{\frac{\theta^k}{1-\lambda_i}}\right)^{\frac{1-\lambda_i}{\theta^k}} \tag{C.4}$$

$$(\pi^k_{\chi|ij})' = \frac{\left((B^k_{ij\chi})'\right)^{\frac{\theta^k}{1-\lambda_i}}}{\left((B^k_{ijV})'\right)^{\frac{\theta^k}{1-\lambda_i}} + \left((B^k_{ijO})'\right)^{\frac{\theta^k}{1-\lambda_i}}} \tag{C.5}$$

$$\widehat{\pi^k_{ij}} = (\widehat{d^k_{ij}}\widehat{w_i})^{-\theta^k}(\widehat{B^k_{ij}})^{\theta^k}/\widehat{\Phi^k_j} \tag{C.6}$$

$$\widehat{\Phi^k_j} = \sum_{i=1}^{J}\pi^k_{ij}(\widehat{d^k_{ij}}\widehat{w_i})^{-\theta^k}(\widehat{B^k_{ij}})^{\theta^k} \tag{C.7}$$

$$(\Upsilon^k_j)' = \sum_{i=1}^{J}\sum_{\chi=\{V,O\}}\frac{(\zeta^k_{ij})'}{(\zeta^k_{ij\chi})'}(\pi^k_{ij})'(\pi^k_{\chi|ij})' \tag{C.8}$$

$$(\varpi_j)' = 1-(1-\alpha)\sum_{k=1}^{K}\frac{\rho\eta^k}{\rho^k}\left(1-(1-\rho^k)/(\Upsilon^k_j)'\right) \tag{C.9}$$

$$(E_j)' = \frac{\widehat{w_j}w_j\bar{L}_j + \widehat{s_j}s_j\bar{K}_j + D_j}{1-\frac{1-\rho}{1-\rho(1-\alpha)}(\varpi_j)'} \tag{C.10}$$

$$\begin{aligned}\widehat{w_j}w_j\bar{L}_j = {} & \frac{\alpha\rho(\varpi_j)'}{1-\rho(1-\alpha)}(E_j)' \\ & + (1-\alpha)\rho\sum_{k=1}^{K}\sum_{m=1}^{J}\sum_{\chi\in\{V,O\}}\eta^k(1-\alpha^k)\frac{(E_m)'}{(\Upsilon^k_m)'}(\pi^k_{jm})'(\pi^k_{\chi|jm})'\left((\mu^k_{xjm})' + (1-(\mu^k_{xjm})')(1-(\beta^k_{jm\chi})')\frac{(\zeta^k_{jm})'}{(\zeta^k_{jm\chi})'}\right)\end{aligned} \tag{C.11}$$

$$\widehat{s_j}s_j\bar{K}_j = (1-\alpha)\rho\sum_{k=1}^{K}\sum_{i=1}^{J}\sum_{\chi\in\{V,O\}}\eta^k\alpha^k\frac{(E_j)'}{(\Upsilon^k_j)'}(\pi^k_{ij})'(\pi^k_{\chi|ij})'\left((\mu^k_{hij})' + (1-(\mu^k_{hij})')(\beta^k_{ij\chi})'\frac{(\zeta^k_{ij})'}{(\zeta^k_{ij\chi})'}\right) \tag{C.12}$$

To operationalize the above system of equations, we require values for the parameters ρ, α, η^k, ρ^k, α^k, θ^k, and λ_i, as well as information on π^k_{ij}, $w_j\bar{L}_j$, $s_j\bar{K}_j$, and D_j.

The values we calibrate the aggregate parameters ρ, α, and η^k to are described in the main paper at the start of Section 5. For α^k, θ^k, and λ_i, we base these on our estimation results; the values of α^k and θ^k are reported in Table 2, while the values for the λ_i's are reported in Table B.4. The ρ^k's are inferred from the Soderbery (2015) elasticities and the α^k values, as discussed in Appendix B.3; the ρ^k values are reported in Table 2. Turning to the information needed on π_{ij}^k, $w_j\bar{L}_j$, $s_j\bar{K}_j$, and D_j, we describe below in Appendix C.2 how we back out information on the sourcing probabilities π_{ij}^k from data on trade shares, while we describe in Appendix C.3 how we obtain the values of $w_j\bar{L}_j$, $s_j\bar{K}_j$, and D_j from the factor market clearing conditions in the initial equilibrium.

The hat algebra system of equations (C.1)-(C.12) can then be solved for a given set of exogenous shocks to trade costs ($\widehat{d_{ij}^k}$), contracting frictions ($(\mu_{hij}^k)'$, $(\mu_{xij}^k)'$), and/or bargaining shares ($(\beta_{ij\chi}^k)'$), as follows:

1. Given $(\mu_{hij}^k)'$, $(\mu_{hij}^k)'$ and $(\beta_{ij\chi}^k)'$, use equations (C.1)-(C.4) to solve for $(B_{ij\chi}^k)'$ and $(B_{ij}^k)'$.
2. Use equation (C.5) and $(B_{ij\chi}^k)'$ to compute $(\pi_{\chi|ij}^k)'$.
3. Guess a vector of $\widehat{w}_j$'s and $\widehat{s}_j$'s.
4. Conditional on $(B_{ij}^k)'$, $\widehat{d_{ij}^k}$, and the guessed values of $\widehat{w}_j$ and $\widehat{s}_j$, use equation (C.7) to solve for $\widehat{\Phi_j^k}$.
5. Use $\widehat{\Phi_j^k}$ and equation (C.6) to compute $\widehat{\pi_{ij}^k}$ and $(\pi_{ij}^k)'$.
6. With $(\pi_{ij}^k)'$ and $(\pi_{\chi|ij}^k)'$, use equations (C.8) and (C.9) to get $(\Upsilon_j^k)'$ and $(\varpi_j)'$.
7. With $(\varpi_j)'$, use equation (C.10) to solve for $(E_j)'$.
8. With all the above information, compute the right-hand sides of equations (C.11) and (C.12), and hence solve for the implied values of $\widehat{w_j}$ and $\widehat{s}_j$ (by dividing by $w_j\bar{L}_j$ and $s_j\bar{K}_j$ respectively).
9. Update $(\widehat{w_j}, \widehat{s}_j)$ with these new values obtained in step 8, and iterate from step 3 until convergence.

This yields counterfactual values in particular for: $\widehat{w_j}$, $\widehat{s}_j$, $\widehat{E_j}$, $\widehat{\pi_{ij}^k}$, $(\pi_{\chi|ij}^k)'$, $(\Upsilon_j^k)'$, $(\varpi_j^k)'$, and $\widehat{\Phi_j^k}$.

C.2 Inferring π_{ij}^k from the observed trade shares

The sourcing probability π_{ij}^k in equation (19) is equal to the share of input varieties from industry k that are sourced by country j from country i. In Eaton and Kortum (2002), π_{ij}^k would also be the share of trade by value in industry k that is imported by country j from country i, as the distribution of prices that country j faces is independent of the identity of the source country i when prices are set competitively. In our setting however, the interaction between a final-good firm and its suppliers is instead mediated by a bargaining process. We therefore need an additional step to map π_{ij}^k to the corresponding trade share by value.

Recall that the expression for trade flows from country i to j in industry k under organizational mode χ is given in equation (30) in the main text. Summing across organizational modes, we have:

$$t_{ij}^k = \sum_{\chi=\{V,O\}} t_{ij\chi}^k = (1-\alpha)\rho\eta^k(1-\alpha^k)\frac{E_j}{\Upsilon_j^k\Phi_j^k}T_i^k(w_i)^{-\theta^k}\left(B_{ij}^k\right)^{-\frac{\theta^k\lambda_i}{1-\lambda_i}}\left(d_{ij}^k\right)^{-\theta^k}$$
$$\times \sum_{\chi\in\{V,O\}}\left(\mu_{xij}^k+(1-\mu_{xij}^k)(1-\beta_{ij\chi}^k)(\zeta_{ij}^k/\zeta_{ij\chi}^k)\right)\left(B_{ij\chi}^k\right)^{\frac{\theta^k}{1-\lambda_i}}.$$

Define ς_{ij}^k to be the following correction term:

$$\varsigma_{ij}^k = \sum_{\chi\in\{V,O\}}\left(\mu_{xij}^k+(1-\mu_{xij}^k)(1-\beta_{ij\chi}^k)(\zeta_{ij}^k/\zeta_{ij\chi}^k)\right)\left(B_{ij\chi}^k/B_{ij}^k\right)^{\frac{\theta^k}{1-\lambda_i}}. \tag{C.13}$$

Note that $\varsigma_{ij}^k = 1$ in the special case of full contractibility ($\mu_{hij}^k = \mu_{xij}^k = 1$). We now have:

$$\frac{t_{ij}^k/\varsigma_{ij}^k}{\sum_{i'=1}^J t_{i'j}^k/\varsigma_{i'j}^k} = \frac{T_i^k(w_id_{ij}^k)^{-\theta^k}(B_{ij}^k)^{-\frac{\theta^k\lambda_i}{1-\lambda_i}}(B_{ij}^k)^{\frac{\theta^k}{1-\lambda_i}}}{\sum_{i'=1}^J T_{i'}^k(w_{i'}d_{i'j}^k)^{-\theta^k}(B_{i'j}^k)^{-\frac{\theta^k\lambda_{i'}}{1-\lambda_{i'}}}(B_{i'j}^k)^{\frac{\theta^k}{1-\lambda_{i'}}}} = \frac{T_i^k(w_id_{ij}^k)^{-\theta^k}(B_{ij}^k)^{\theta^k}}{\sum_{i'=1}^J T_{i'}^k(w_{i'}d_{i'j}^k)^{-\theta^k}(B_{i'j}^k)^{\theta^k}} = \pi_{ij}^k.$$

It follows from the above that through the structure of our model, we can recover the sourcing probabilities, π_{ij}^k, by dividing observed bilateral trade flows in the data by the corresponding correction term in (C.13). After we have estimated the model, the ς_{ij}^k correction term can be evaluated exactly in the baseline equilibrium. When we perform the hat algebra counterfactuals, where we need the initial values of π_{ij}^k as key sufficient statistics to evaluate general equilibrium responses, we now first apply the correction terms in order to back out π_{ij}^k, instead of reading this off directly from the entries of the ICIO tables.

C.3 Computing $w_j\bar{L}_j$, $s_j\bar{K}_j$, and D_j

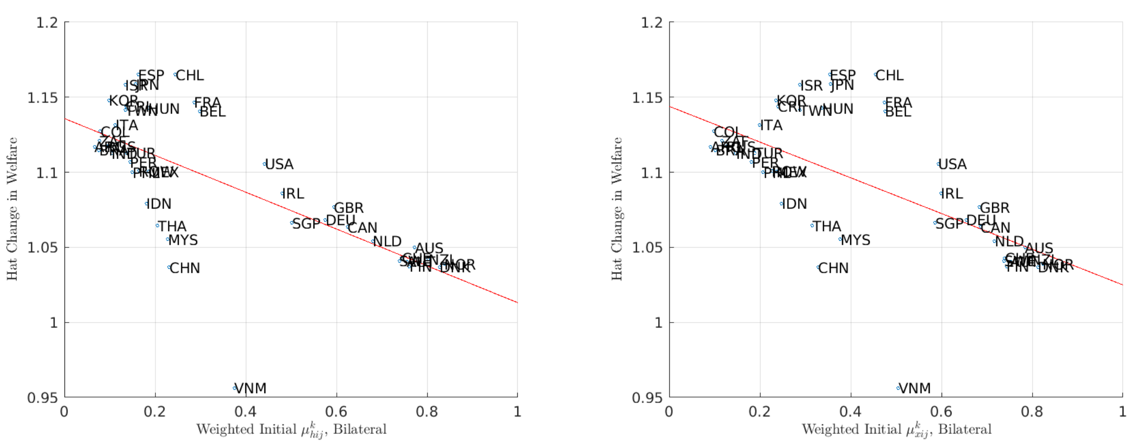
We require values for $w_j\bar{L}_j$, $s_j\bar{K}_j$, and D_j – total factor payments to labor, to capital, and the trade deficit, respectively – in the initial equilibrium, to operationalize the hat algebra system of equations. We recover these values from the factor market clearing conditions in the initial equilibrium as follows. First, after estimating the model, we compute values for μ_{hij}^k, μ_{xij}^k, and $\beta_{ij\chi}^k$ for the baseline equilibrium as implied by our estimates. We plug these values into the analogue of equations (C.1)-(C.5), (C.8) and (C.9) written in levels (rather than in changes), namely equations (A.26)-(A.30), (A.33) and (A.34) presented earlier in Section A.3. Together with observed values of π_{ij}^k obtained from the ICIO data (after applying the correction term ς_{ij}^k to the trade values), this allows us to compute ζ_{ij}^k, $\zeta_{ij\chi}^k$, $B_{ij\chi}^k$, B_{ij}^k, $\pi_{\chi|ij}^k$, Υ_j^k, and ϖ_j^k in the initial equilibrium.

We then work with the analogue of equations (C.10), (C.11), and (C.12) in levels, i.e., equations (A.35), (A.36), and (A.37). Given data on country expenditures – the E_j's – from the ICIO data, (A.35)-(A.37) describes a system of $3J$ equations that is linear in the $3J$ unknowns: $\{w_j\bar{L}_j, s_j\bar{K}_j, D_j\}$, for $j \in \{1, \cdots, J\}$. We can solve this system of equations and thus infer the levels of $w_j\bar{L}_j, s_j\bar{K}_j$, and D_j in the baseline equilibrium. These $3J$ initial values are what we feed into the hat algebra system when evaluating any counterfactual scenario.

C.4 Additional Details on Counterfactuals: Removing Contracting Frictions

We report additional details on the counterfactuals conducted in Section 5.1 in the main paper, in which we examine the implications of moving to full contractibility (where $\mu_{hij}^k = 1$ and $\mu_{xij}^k = 1$, for all i, j, and k).

Welfare change and initial contractibility: Figure C.1 confirms that countries that start with lower average contractibility – in either headquarter or supplier input tasks – reap more substantial gains in welfare in the full contractibility world.



(a) Initial μ_{hij}^k (b) Initial μ_{xij}^k

Figure C.1: Full Contractibility Counterfactual: Changes in Welfare against Initial Contractibility

Notes: The figures plot the "hat" changes in country welfare against the initial average levels of μ_{hij}^k and μ_{xij}^k in each country. The welfare changes are from the full contractibility counterfactual, in which all μ_{hij}^k's and μ_{xij}^k's are set to 1.

Note that the vertical axis in Figure C.1 illustrates the "hat" change in country welfare ($\widehat{U_j}$) under full contractibility relative to the baseline world with contracting frictions (from Row 1, Table 3(a)). On the other hand, the horizontal axes illustrate the initial average values of μ^k_{hij} and μ^k_{xij} respectively in the baseline world; we take a weighted average of μ^k_{hij} (respectively, μ^k_{xij}) over all observations where the country is either an importer or exporter, using the associated trade flow values in the baseline equilibrium as weights.

Decomposing the change in country contracting capacities: The results in Row 1, Table 3(a) in the main paper show that the most important source of welfare gains from removing contracting frictions stems from changes in countries' contracting capacities, namely the $\prod_{k=1}^K (\widehat{B^k_{jj}})^{\eta^k(1-\alpha^k)(1-\alpha)}$ term. To further unpack the underlying sources of variation, we take logarithms of this term and rewrite it as:

$$\log\left(\prod_{k=1}^K \left(\widehat{B^k_{jj}}\right)^{\eta^k(1-\alpha^k)(1-\alpha)}\right) = \sum_{k=1}^K \eta^k(1-\alpha^k)(1-\alpha)\log\left(\widehat{B^k_{jj}}\right) = (1-\alpha)\sum_{k=1}^K \widetilde{\eta}^k \times \widetilde{B}^k_j,$$

where we introduce the notation: $\widetilde{\eta}^k \equiv \eta^k(1-\alpha^k)$ and $\widetilde{B}^k_j \equiv \log(\widehat{B^k_{jj}})$ to ease the exposition. Note further that:

$$\begin{aligned}(1-\alpha)\sum_{k=1}^K \widetilde{\eta}^k \times \widetilde{B}^k_j &= (1-\alpha)\sum_{k=1}^K \left(\widetilde{\eta}^k - \bar{\widetilde{\eta}} + \bar{\widetilde{\eta}}\right) \times \left(\widetilde{B}^k_j - \bar{\widetilde{B}}_j + \bar{\widetilde{B}}_j\right) \\ &= (1-\alpha)\sum_{k=1}^K \left[\bar{\widetilde{\eta}}\bar{\widetilde{B}}_j + \bar{\widetilde{B}}_j\left(\widetilde{\eta}^k - \bar{\widetilde{\eta}}\right) + \bar{\widetilde{\eta}}\left(\widetilde{B}^k_j - \bar{\widetilde{B}}_j\right) + \left(\widetilde{\eta}^k - \bar{\widetilde{\eta}}\right)\left(\widetilde{B}^k_j - \bar{\widetilde{B}}_j\right)\right] \\ &= (1-\alpha)K\bar{\widetilde{\eta}}\bar{\widetilde{B}}_j + (1-\alpha)\sum_{k=1}^K \left(\widetilde{\eta}^k - \bar{\widetilde{\eta}}\right)\left(\widetilde{B}^k_j - \bar{\widetilde{B}}_j\right), \qquad \text{(C.14)}\end{aligned}$$

where: $\bar{\widetilde{\eta}} \equiv (1/K)\sum_{k=1}^K \eta^k(1-\alpha^k)$ and $\bar{\widetilde{B}}_j \equiv (1/K)\sum_{k=1}^K \log(\widehat{B^k_{jj}})$ are the averages across industries of the exponents and the "hat" changes in contracting capacities, respectively. The decomposition in equation (C.14) has a natural interpretation: The first term $(1-\alpha)K\bar{\widetilde{\eta}}\bar{\widetilde{B}}_j$ captures the welfare impacts that can be attributed to the average (log) change in industry contracting capacity ($\bar{\widetilde{B}}_j$) multiplied by the average welfare elasticity ($(1-\alpha)\bar{\widetilde{\eta}}$). The second term picks up the covariance between these two forces.

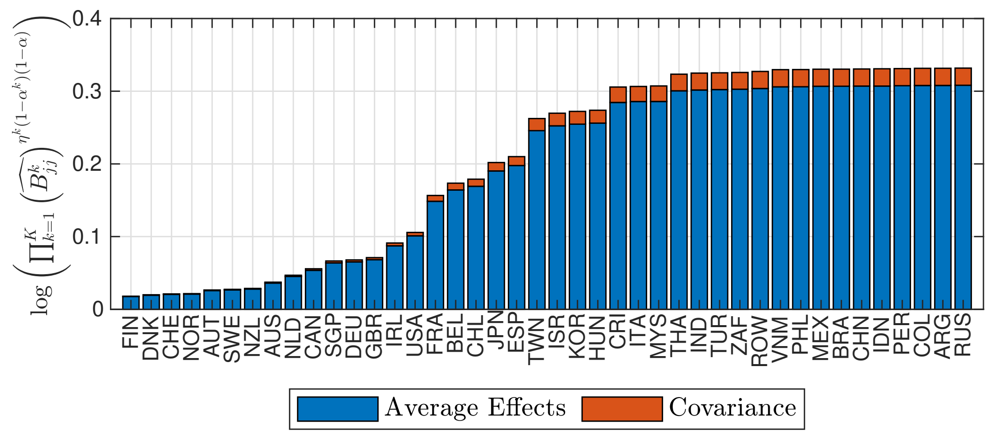
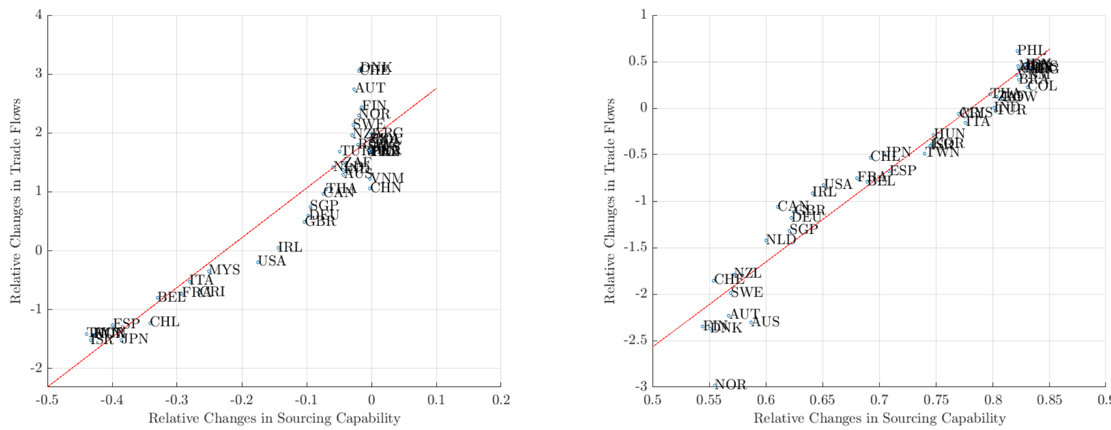


Figure C.2: Decomposing the Welfare Impacts of the Contracting Capacities Term

Notes: This figure presents the decomposition of welfare impacts attributable to shifts in contracting capacities, based on equation (C.14), for the full contractibility counterfactual. "Average Effects" (blue bar) illustrates the $(1-\alpha)K\bar{\widetilde{\eta}}\bar{\widetilde{B}}_j$ term, while "Covariance" (red bar) illustrates the $(1-\alpha)\sum_{k=1}^K(\widetilde{\eta}^k - \bar{\widetilde{\eta}})(\widetilde{B}^k_j - \bar{\widetilde{B}}_j)$ term.

Figure C.2 illustrates this decomposition country-by-country for the full contractibility counterfactual. This shows that in each country, the average effect term (rather than the covariance) dominates quantitatively, explaining more than 90% of the variation in the log of $\prod_{k=1}^{K}(\widehat{B_{jj}^{k}})^{\eta^{k}(1-\alpha^{k})(1-\alpha)}$. Note that the average elasticity across industries $(1-\alpha)\bar{\bar{\eta}}$ is equal to 0.49; since this does not differ across countries, the variation across countries j in the height of the "average effect" term (blue bars) is driven by differences in $\bar{\bar{\widehat{B}}}_{j}$ (i.e., the average "hat" change in contracting capacities across industries in country j).

Responses of trade flows: In Figure C.3 below, we illustrate how trade flows respond when contracting frictions are removed. We find that changes in countries' trade flows with China relative to the US are driven by changes in their relative contracting capacities with respect to these two countries.



(a) Relative Imports

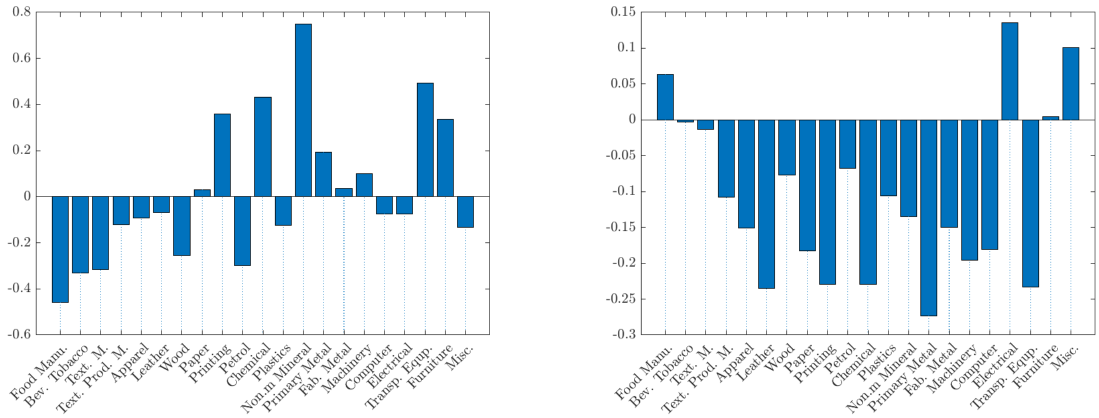
(b) Relative Exports

Figure C.3: Hat Changes in Trade Patterns by Country, vis-à-vis China and the USA

Notes: The two panels present the relative changes in trade flows against the relative changes in contracting capacities vis-à-vis China and the US, in the full contractibility counterfactual. On the vertical axis, the relative change in country j's imports in Panel (a) is given by: $(1/K)\sum_{k}(\widehat{t_{\mathrm{CHN},j}^{k}-t_{\mathrm{USA},j}^{k}})$, and the relative change in country i's exports in Panel (b) is given by: $(1/K)\sum_{k}(\widehat{t_{i,\mathrm{CHN}}^{k}-t_{i,\mathrm{USA}}^{k}})$. On the horizontal axis, the "relative changes in contracting capacity" in the two panels are respectively: $(1/K)\sum_{k}(\widehat{B_{\mathrm{CHN},j}^{k}}-\widehat{B_{\mathrm{USA},j}^{k}})$ and $(1/K)\sum_{k}(\widehat{B_{i,\mathrm{CHN}}^{k}}-\widehat{B_{i,\mathrm{USA}}^{k}})$.

Panel (a) shows that the "hat" change in countries' imports from China relative to their imports from the US, $(1/K)\sum_{k}(\widehat{t_{\mathrm{CHN},j}^{k}}-\widehat{t_{\mathrm{USA},j}^{k}})$, is positively correlated with the "hat" change in their contracting capacities when importing from China relative to the US, as captured by $(1/K)\sum_{k}(\widehat{B_{\mathrm{CHN},j}^{k}}-\widehat{B_{\mathrm{USA},j}^{k}})$. Put otherwise, countries that see a bigger increase in their contracting capacities when importing from China compared to the US indeed see a larger rise in their imports from China relative to the US. Analogously, Panel (b) shows that the "hat" change in countries' exports to China relative to their exports to the US, $(1/K)\sum_{k}(\widehat{t_{i,\mathrm{CHN}}^{k}}-\widehat{t_{i,\mathrm{USA}}^{k}})$, is positively correlated with the shift in China's contracting capacities relative to the US' over these respective countries, $(1/K)\sum_{k}(\widehat{B_{i,\mathrm{CHN}}^{k}}-\widehat{B_{i,\mathrm{USA}}^{k}})$.

In Figure C.4, we focus on the US and China, and examine shifts in their specialization patterns as reflected in the composition of their industry-level exports in the full contractibility counterfactual. In the case of China (Panel (a)), the industries with the largest export growth (in percentage terms) are: Non-metallic minerals, Transportation equipment, Chemicals, Printing, and Furniture. For the US (Panel (b)), the industries that experience the largest percentage increase in exports are: Electrical, Miscellaneous, and Food Manufacturing.



(a) China (b) US

Figure C.4: Hat Changes in Industry-level Exports

Notes: The panels plot the "hat" changes in industry-level exports for China and the US, in the full contractibility counterfactual.

Uncorrelated supplier draws: Figure C.5 compares the "hat" changes in welfare when moving to full contractibility under two scenarios: (i) the nested Fréchet draws are set to be uncorrelated within source-country nests (i.e., $\lambda_i = 0$ for all i); and (ii) these correlation parameters take on their estimated values for the three country groups (i.e., $\lambda_{(1)} = 0.944$, $\lambda_{(2)} = 0.923$, $\lambda_{(3)} = 0.852$). Scenario (i) is the counterfactual reported in Row 4, Table 3(a) in the main paper, while scenario (ii) is the baseline counterfactual reported in Row 1, Table 3(a). The figure shows that the welfare gain of countries with low initial rule of law would tend to be over-stated by a greater extent had we assumed a zero-correlation structure in the supplier draws. Specifically, the vertical axis plots the difference in the "hat" change in welfare between scenarios (i) and (ii). This is negatively correlated with the country rule of law index arrayed on the horizontal axis.

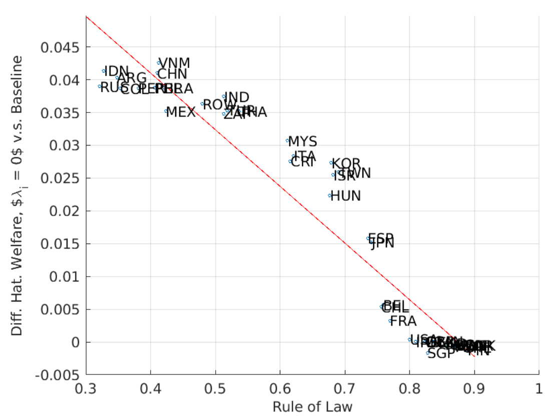


Figure C.5: Hat Changes in Welfare, Uncorrelated Productivity Draws versus Baseline Counterfactual

Notes: The vertical axis plots the difference in the "hat" change in welfare under full contractibility, between the scenario in which λ_i is set to 0 for all countries versus a scenario in which the correlation parameters take on their estimated values. This difference is illustrated against initial country rule of law.

C.5 Endogenous Firm Entry

In this appendix section, we provide details and derivations for the model extension in which we allow for the entry of firms, with the mass of firms N_j endogenously pinned down by a free entry condition.

We now assume that in each country j, there are infinitely many potential firms (each with an outside option of zero). To enter the market, each potential firm incurs a fixed cost f_j that is denominated in units of the same CES aggregate over final goods, $(\int_{\omega\in\Omega} q_j(\omega)^\rho d\omega))^{\frac{1}{\rho}}$. After these fixed costs are incurred, the entrant draws its core productivity ϕ from the distribution $G_j(\phi)$, and then starts producing. Production itself does not require additional fixed costs, and so all entrants will choose to commence production (regardless of the realized value of ϕ). In equilibrium, the mass of firms, N_j, will adjust so that the expected value of entry is zero.

Free entry condition: Recall from Section 3.1 that, conditional on core productivity ϕ, the (post-assembly) profit of the firm is:

$$F_j(\phi) = \left(\frac{1-\rho}{1-\rho(1-\alpha)}\right)\varpi_j R_j(\phi), \tag{C.15}$$

where the revenue of the firm is given by: $R_j(\phi) = A_j^{1-\rho} q_j(\phi)^\rho$. We next substitute in the expression for $q_j(\phi)$ from (26) and for A_j from (A.25), to obtain:

$$R_j(\phi) = \left(\frac{\phi}{\bar{\phi}_j}\right)^{\frac{\rho}{1-\rho}} \frac{E_j}{N_j}, \tag{C.16}$$

where recall that: $\bar{\phi}_j \equiv (\int_\phi \phi^{\frac{\rho}{1-\rho}} dG_j(\phi))^{\frac{1-\rho}{\rho}}$. Intuitively, the revenue of the firm is increasing in country j's market size (captured by E_j) and decreasing in the mass of firms (N_j).

The free entry condition that must hold in equilibrium is then:

$$\int_\phi F_j(\phi) dG_j(\phi) = P_j f_j,$$

where P_j is the ideal price index associated with the final-good bundle. Substituting in the expression for $F_j(\phi)$ implied by (C.15) and (C.16), and simplifying, we get that the equilibrium mass of firms is:

$$N_j = \left(\frac{1-\rho}{1-\rho(1-\alpha)}\right)\frac{\varpi_j E_j}{P_j f_j}. \tag{C.17}$$

Aggregation: Referring back to the system of equations (A.26)-(A.37) in Appendix A.3 that defines the equilibrium in the baseline model, the one relationship that potentially changes in this extension with free entry is equation (A.35) that pins down aggregate expenditure in each country. We now derive the aggregate expenditure relationship for this extended version of the model with endogenous entry of firms. In country j, the total expenditure on final goods, E_j, is given by:

$$E_j = w_j \bar{L}_j + s_j \bar{K}_j + D_j + P_j N_j f_j. \tag{C.18}$$

This spending comes from three sources: demand for final goods from consumers, that is paid out of factor income, $w_j\bar{L}_j + s_j\bar{K}_j$; that is paid out of the trade deficit, D_j; and demand for final goods to cover the fixed costs of entry $P_j N_j f_j$. Note though that we can take the expression for $P_j N_j f_j$ implied by (C.17) and substitute this into (C.18); a quick manipulation then yields:

$$E_j = \frac{w_j \bar{L}_j + s_j \bar{K}_j + D_j}{1 - \frac{1-\rho}{1-\rho(1-\alpha)}\varpi_j}.$$

This aggregate expenditure relationship turns out to be exactly the same as equation (A.35) in the baseline model. This is intuitive since in both the baseline and extended models, the post-assembly profits of firms are channeled towards purchases of final goods; the only difference is that these profits are rebated to households who use it to consume final goods in the baseline model, while these profits are directly expended on the fixed entry costs in the extended model.

The system of equations that defines the equilibrium in the extended model with free entry thus comprises (A.26)-(A.37) as in the baseline model, augmented with an equation for the endogenous mass of firms in each country, N_j. We have already seen an expression for N_j in (C.17), which we will simplify next.

Mass of varieties: The ideal price index, P_j, can be written as:

$$P_j = \left(N_j \int_\phi p_j(\phi)^{-\frac{\rho}{1-\rho}} dG_j(\phi)\right)^{-\frac{1-\rho}{\rho}} = \left(N_j \int_\phi \left(\frac{q_j(\phi)}{A_j}\right)^\rho dG_j(\phi)\right)^{-\frac{1-\rho}{\rho}} = N_j^{-\frac{1-\rho}{\rho}} A_j^{1-\rho} \left(\int_\phi q_j(\phi)^\rho dG_j(\phi)\right)^{-\frac{1-\rho}{\rho}}.$$

We substitute for $q_j(\phi)$ from (26) into the above; taking the resulting expression for P_j, and plugging it into (C.17), we obtain:

$$N_j^{\frac{2\rho-1}{\rho}} = \left(\frac{1-\rho}{1-\rho(1-\alpha)}\right) \frac{\varpi_j E_j}{f_j} \rho\bar{\phi}_j(1-\alpha)^{1-\alpha} \left(\frac{\alpha}{1-\rho(1-\alpha)} \frac{\varpi_j}{w_j}\right)^\alpha \times \prod_{k=1}^K \left[(\alpha^k/s_j)^{\alpha^k}(1-\alpha^k)^{1-\alpha^k}\eta^k(\bar{\Gamma}^k)^{\frac{1-\rho^k}{\rho^k}}(\Phi_j^k)^{\frac{1-\alpha^k}{\theta^k}}(\Upsilon_j^k)^{\frac{1-\rho^k}{\rho^k}}\right]^{\eta^k(1-\alpha)} \quad \text{(C.19)}$$

It is natural to assume that $\rho > 0.5$, in which case the mass of firms will be increasing in aggregate expenditures, E_j, and in the profit share, ϖ_j, while decreasing in the fixed entry costs, f_j. This is satisfied in practice by our choice of $\rho = 0.8$. (If on the other hand $\rho < 0.5$, the equilibrium would no longer be stable, as a small perturbation in the mass of firms around its equilibrium value would either send N_j to zero or to infinity.)

The equilibrium in the version of the model with free entry is thus pinned down by equations (A.26)-(A.37) augmented by equation (C.19). The corresponding hat algebra system of equations for evaluating exact hat counterfactuals is given by equations (C.1)-(C.12) augmented by the following equation (C.20):

$$(\widehat{N_j})^{\frac{2\rho-1}{\rho}} = (\widehat{\varpi_j})^{1+\alpha}(\widehat{E_j})(\widehat{w_j})^{-\alpha} \times \prod_{k=1}^K \left[(\widehat{s_j})^{-\alpha^k}(\widehat{\Phi_j^k})^{\frac{1-\alpha^k}{\theta^k}}(\widehat{\Upsilon_j^k})^{\frac{1-\rho^k}{\rho^k}}\right]^{\eta^k(1-\alpha)} \quad \text{(C.20)}$$

Welfare: The expression for welfare now needs to take into account that some final goods are used to cover the fixed costs of entry; these do not contribute to consumers' utility. From equation (A.35), households' disposable income comprises a share: $(w_j\bar{L}_j + s_j\bar{K}_j + D_j)/E_j = 1 - \frac{1-\rho}{1-\rho(1-\alpha)}\varpi_j$ of aggregate expenditures. Since the CES utility function is homothetic, this implies that welfare is given by the corresponding expression in the baseline model, i.e., equation (27), scaled proportionally by $1 - \frac{1-\rho}{1-\rho(1-\alpha)}\varpi_j$:

$$\begin{aligned} U_j &= \left(1 - \frac{1-\rho}{1-\rho(1-\alpha)}\varpi_j\right) N_j^{\frac{1-\rho}{\rho}} \rho E_j \bar{\phi}_j (1-\alpha)^{1-\alpha} \left(\frac{\alpha}{1-\rho(1-\alpha)} \frac{\varpi_j}{w_j}\right)^\alpha \\ &\quad \times \prod_{k=1}^K \left[(\alpha^k/s_j)^{\alpha^k}(1-\alpha^k)^{1-\alpha^k}\eta^k\bar{\Gamma}^{\frac{1-\rho^k}{\rho^k}}(\Phi_j^k)^{\frac{1-\alpha^k}{\theta^k}}(\Upsilon_j^k)^{\frac{1-\rho^k}{\rho^k}}\right]^{\eta^k(1-\alpha)} \\ &= N_j f_j \left(\frac{1-\rho(1-\alpha)}{(1-\rho)\varpi_j} - 1\right). \end{aligned}$$

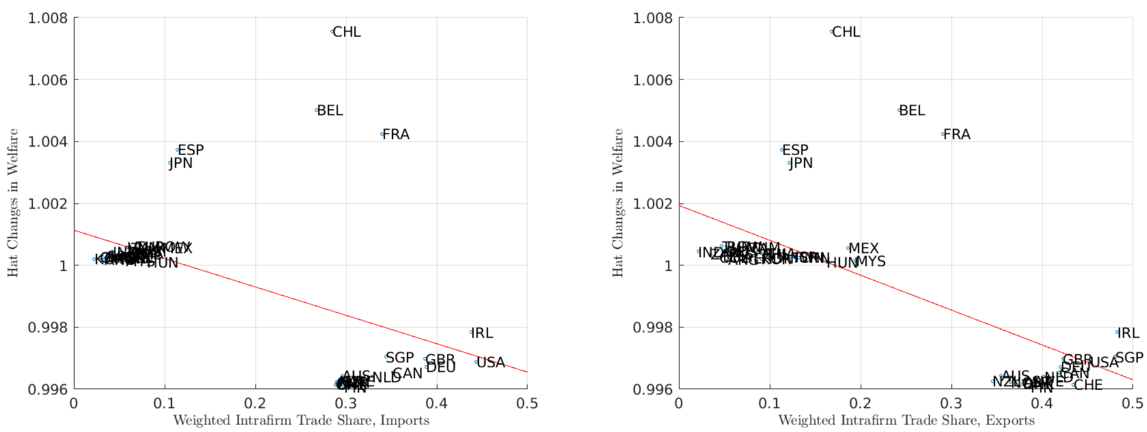
Note that this last expression is derived by substituting in from (C.19). From this, one can see that a higher profit share, ϖ_j, has two opposing effects on welfare: It tends to raise utility through a love-of-variety effect (i.e., raising N_j). On the other hand, a higher ϖ_j means that more resources are diverted toward firm entry rather than consumption.

Based on the above, we can evaluate "hat" changes in welfare (e.g., for the counterfactual in Row 5 of Table 3(a)) via:

$$\widehat{U_j} = \widehat{N_j}\left(\frac{1-\rho(1-\alpha) - (1-\rho)\widehat{\varpi_j}\varpi_j}{1-\rho(1-\alpha)-(1-\rho)\varpi_j}\right)\frac{1}{\widehat{\varpi_j}}.$$

C.6 Additional Details on Counterfactuals: Gains from Integration

Figure C.6 illustrates the welfare changes for the "gains from vertical integration" counterfactual exercise from Row 7 of Table 3(a), as discussed in Section 5.2 in the main paper. The vertical axis plots the "hat" change in welfare when δ_{ij}^{k} is set to 1 country-by-country, for each country j. This is illustrated against the weighted-average intrafirm import share (Panel (a)) and intrafirm export share (Panel (b)) for country j in the initial equilibrium. Figure C.6 verifies that countries tend to experience a larger welfare loss if they were initially more reliant on integration in their sourcing modes, as reflected through these initial intrafirm trade shares. The correlation between the illustrated variables is -0.47 in Panel (a) and -0.65 in Panel (b). While all countries experience a decrease in their contracting capacities when integration is removed as an organizational mode (i.e., $\widehat{B_{jj}^{k}} < 0$), some countries do see an increase in welfare as a result of general equilibrium factor price and expenditure effects.



(a) Initial intrafirm import shares

(b) Initial intrafirm exports

Figure C.6: Vertical Integration Counterfactual: Changes in Welfare against Initial Intrafirm Trade

Notes: The figures plot the "hat" changes in country welfare against the weighted-average initial intrafirm import and intrafirm export shares respectively (where the weights are the initial values of the associated trade flows). The welfare changes are from the gains from integration counterfactual, in which we set $\delta_{ij}^{k} = 1$ country-by-country.